PINNs model validation using shell elements

Astraea Software Co., Ltd

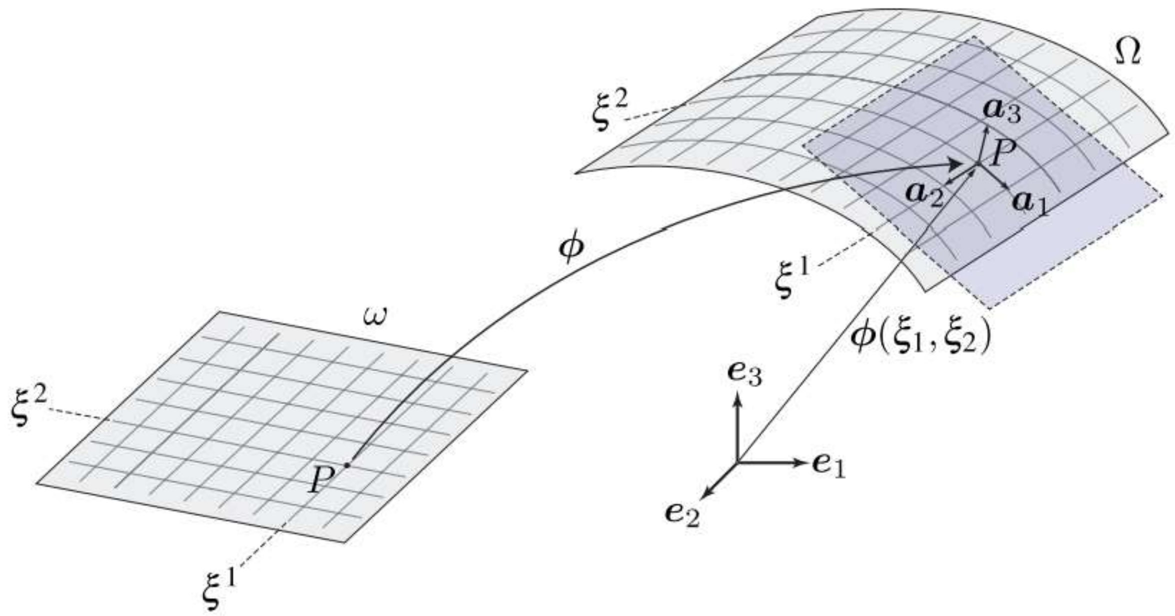
December 25, 2024

Rev.1

Outline

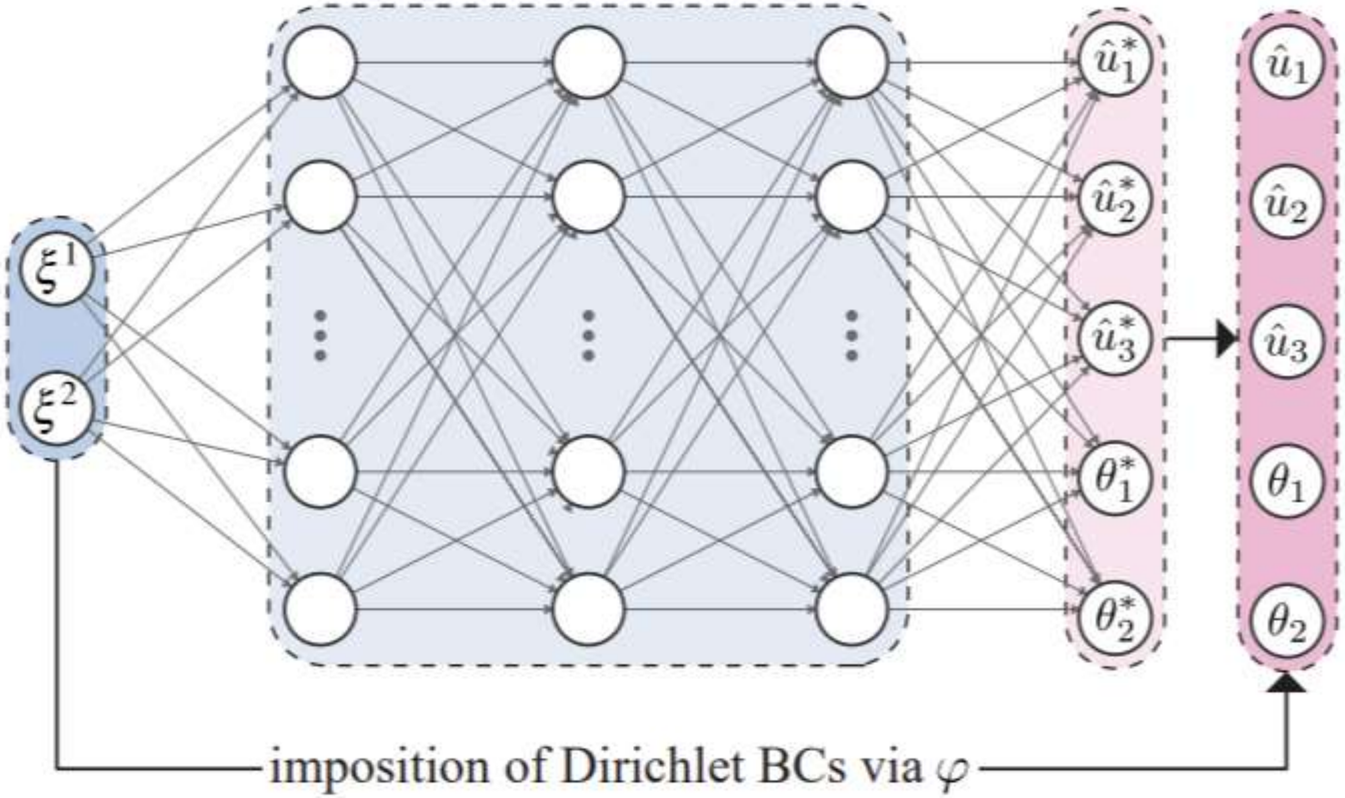
- The PINNs models verified up until now have only X, Y, and Z translational degrees of freedom for the input points and are AI models that assume so-called FEM solid elements.
- However, in actual FEM analysis, shell elements are also widely used, which have five degrees of freedom for input: θx and θy rotation in addition to X, Y, and Z translation.
- Therefore, when applying PINNs to a surrogate model that uses these shell elements, it is necessary to apply a PINNs model that has five degrees of freedom for input.
- This time, we verified a PINNs model that follows the shell element formulation described below.

Shell Element Formulation



The mid-plane of the shell is defined based on a chart Φ that maps from the reference domain ω to the physical domain Ω.
We referred to the formulations in the papers below.

Reference from *Physics-Informed Neural Networks for Shell Structures,* Mechanics & Materials Lab, Department of Mechanical and Process Engineering, ETH Zurich, 8092 Zurich, Switzerland



PINN predicts the scaled global deformation $\hat{u}^*$ and rotation θ^* , which it multiplies by the trial function φ to impose Dirichlet boundary conditions.

The loss function for training the network is defined below based on the weak form.

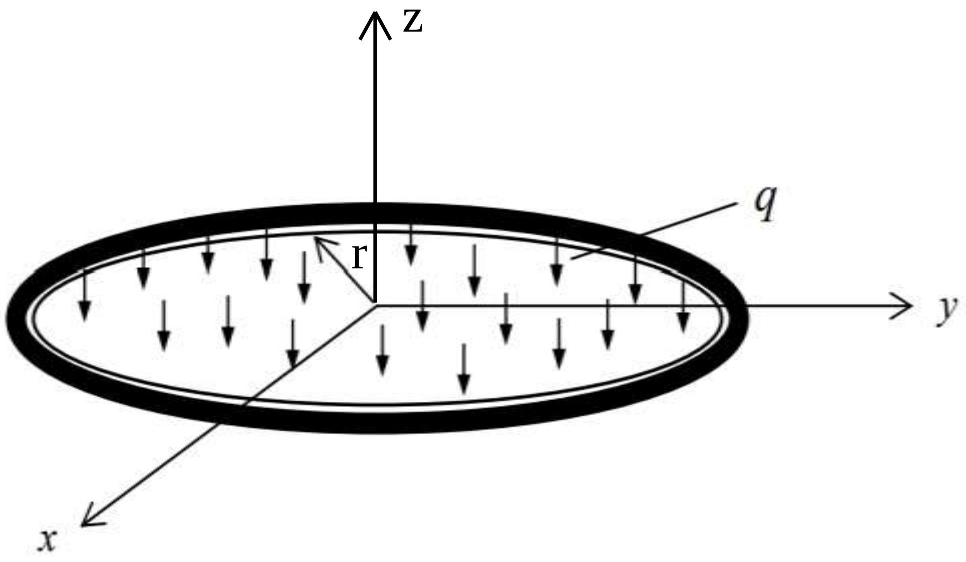
$$\mathcal{L}_{\text{weak}}(\tau) = \frac{|\omega|}{N_c} \sum_{i=1}^{N_c} \Big(\underbrace{\frac{1}{2} t \boldsymbol{e}_\tau(\boldsymbol{\xi}_i) : \mathbb{C}(\boldsymbol{\xi}_i) : \boldsymbol{e}_\tau(\boldsymbol{\xi}_i)}_{\text{membrane energy}} + \underbrace{\frac{1}{2} \frac{t^3}{12} \boldsymbol{k}_\tau(\boldsymbol{\xi}_i) : \mathbb{C}(\boldsymbol{\xi}_i) : \boldsymbol{k}_\tau(\boldsymbol{\xi}_i)}_{\text{bending energy}}$$
$$+ \underbrace{\frac{1}{2} \kappa t \boldsymbol{\gamma}_\tau(\boldsymbol{\xi}_i) \cdot \mathbb{D}(\boldsymbol{\xi}_i) \cdot \boldsymbol{\gamma}_\tau(\boldsymbol{\xi}_i)}_{\text{shear energy}} - \underbrace{W_{\text{ext},\tau}(\boldsymbol{\xi}_i)}_{\text{external work}} \Big) \sqrt{a(\boldsymbol{\xi}_i)},$$

Disk Parametric Training

Outline

- Verification of the shell element PINNs model was carried out using a disk model.
- The disk model is a circle as shown on the next page and is fixed all along its arc. The displacement and stress values are determined when a uniformly distributed load is applied to it.
- For training, 50 disk models with radii randomly selected in the range of 0.1m to 0.5m were used. This aimed to create a parametric model that could handle any radius value within this range. The theoretical solutions for displacement and stress for the same model were also referenced during training.
- The predicted results showed an excellent agreement for the unknown shape when compared with the theoretical solutions.

Verification model



$E = 10.92e11 N/m^2$
$\nu = 0.3$
$q = 6.895e6\ N/m^2$
Thickness = 0.0254m

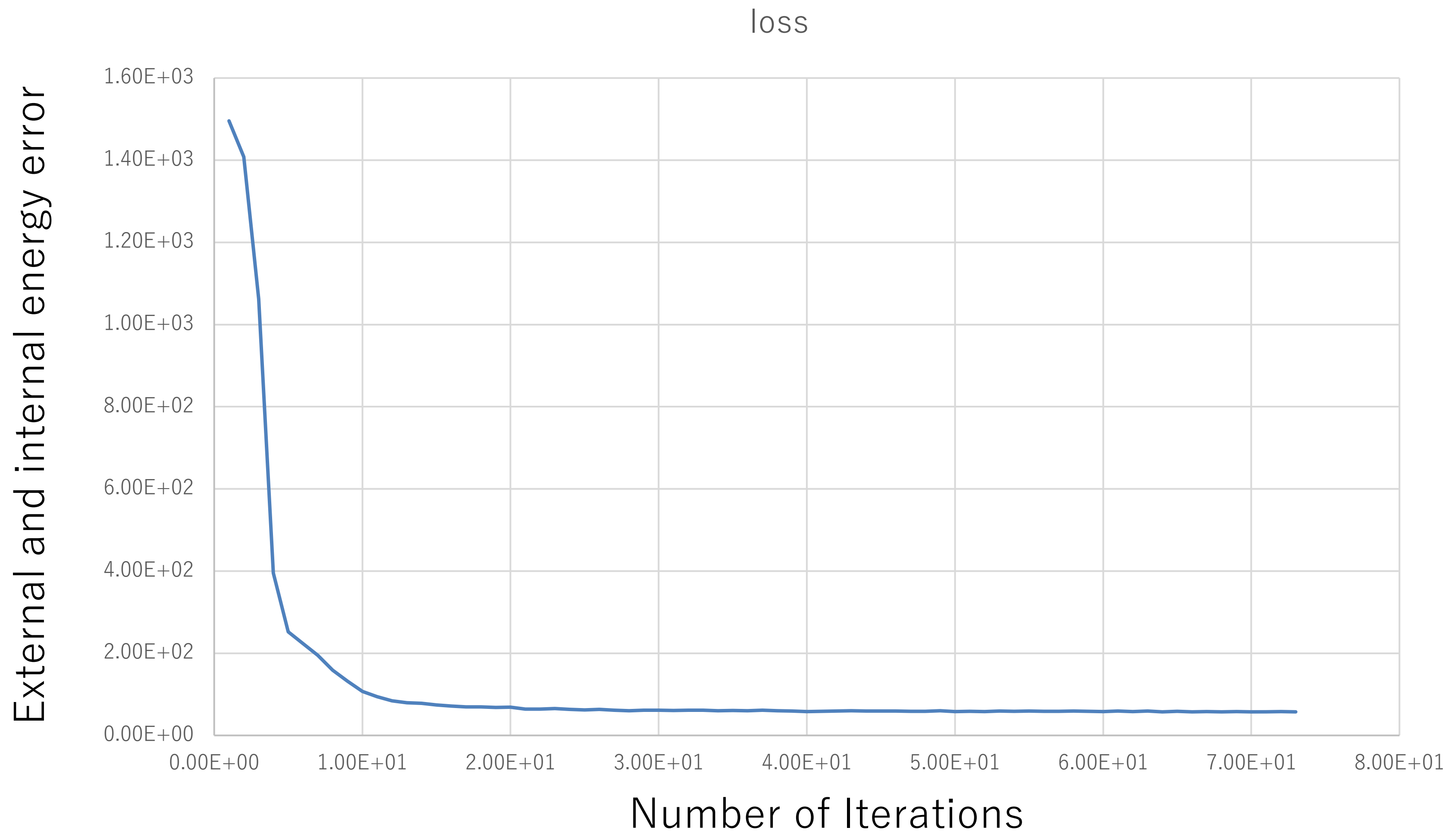
The circular plate is fixed at all boundary edges and a distributed load of 6.895e3 N/m^2 is applied to the plate.

Young's modulus: E = 10.92e11N/m^2
Poisson's ratio: 0.3

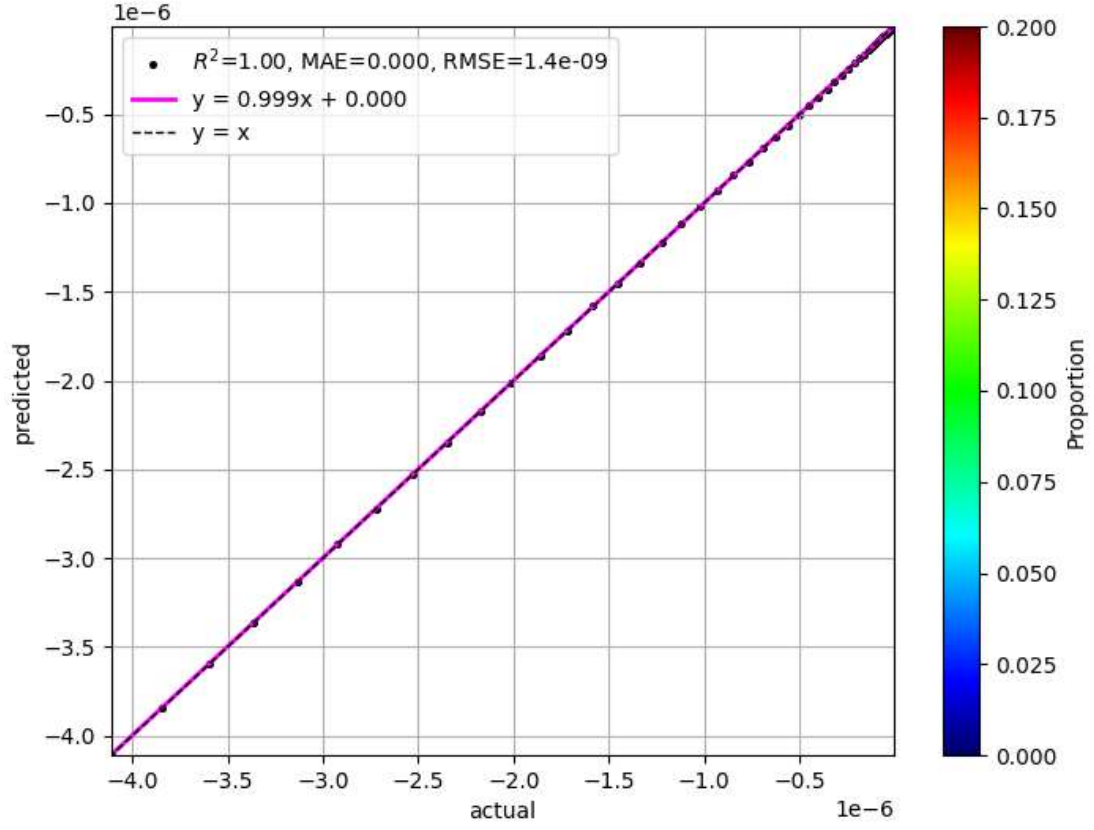
The plate thickness is fixed at 0.0254m and the radius is randomly selected from 0.1 to 0.5m.
Thick shell elements are applied.

Displacement prediction model verification

Convergence of loss function

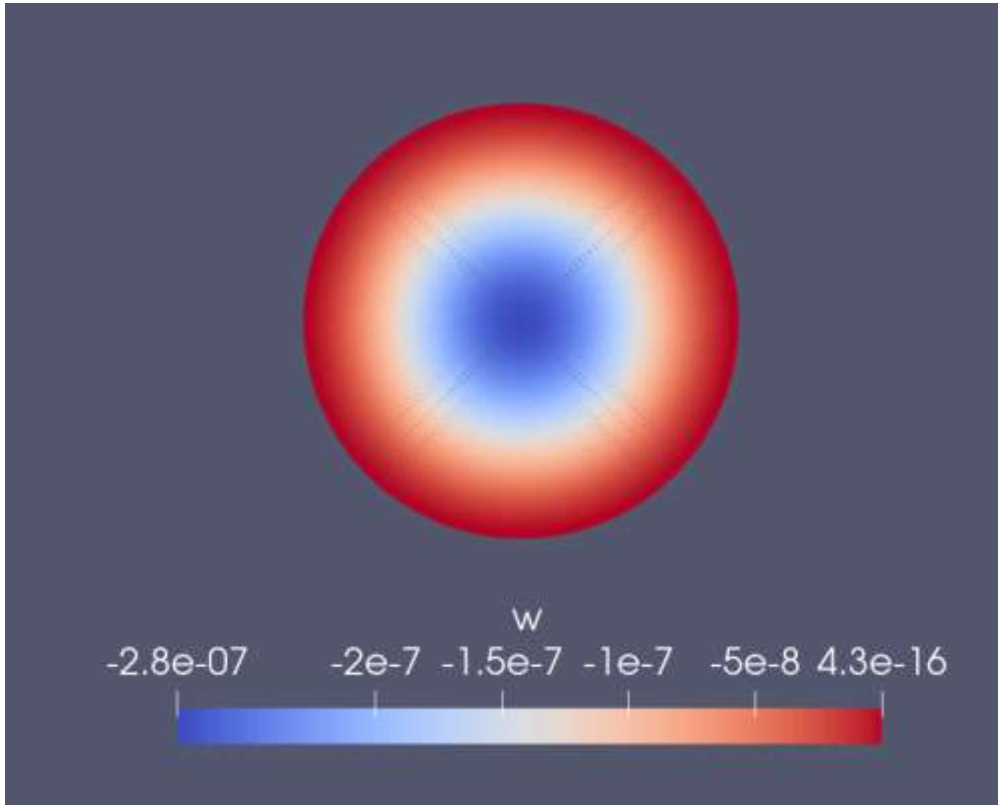


Z-displacement precision plot at the center of the disk (x=0, y=0)

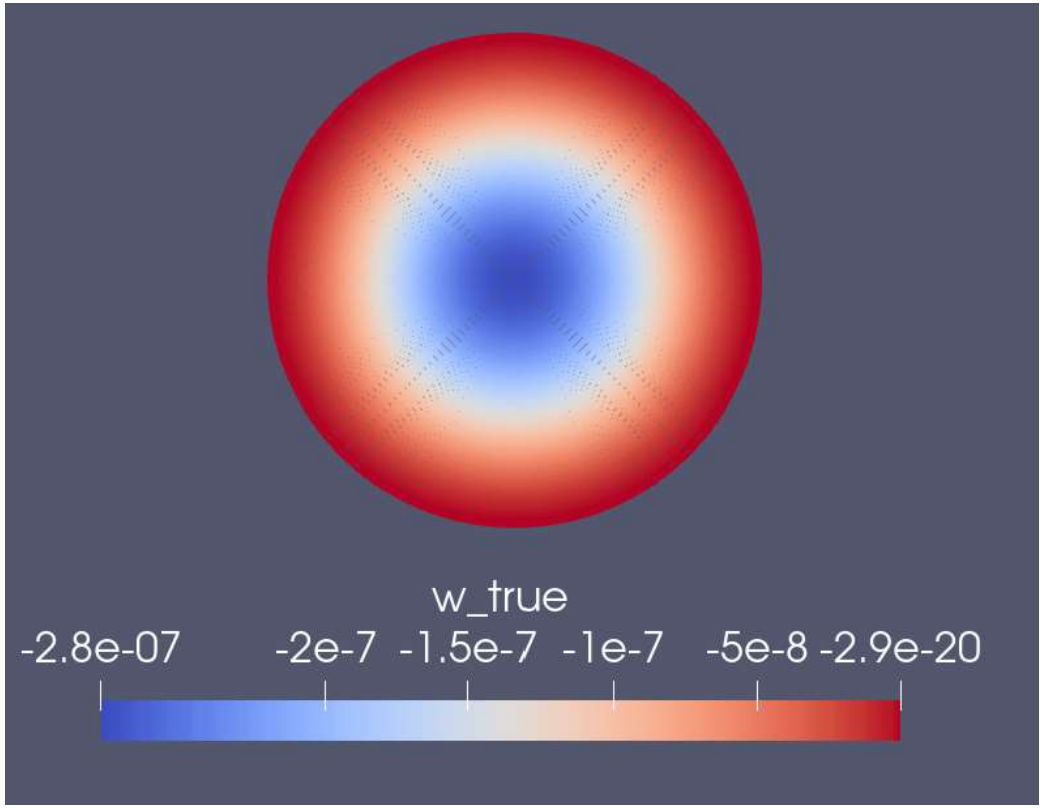


Z-direction displacement distribution
Radius: 0.255102m (known shape)

PINNs

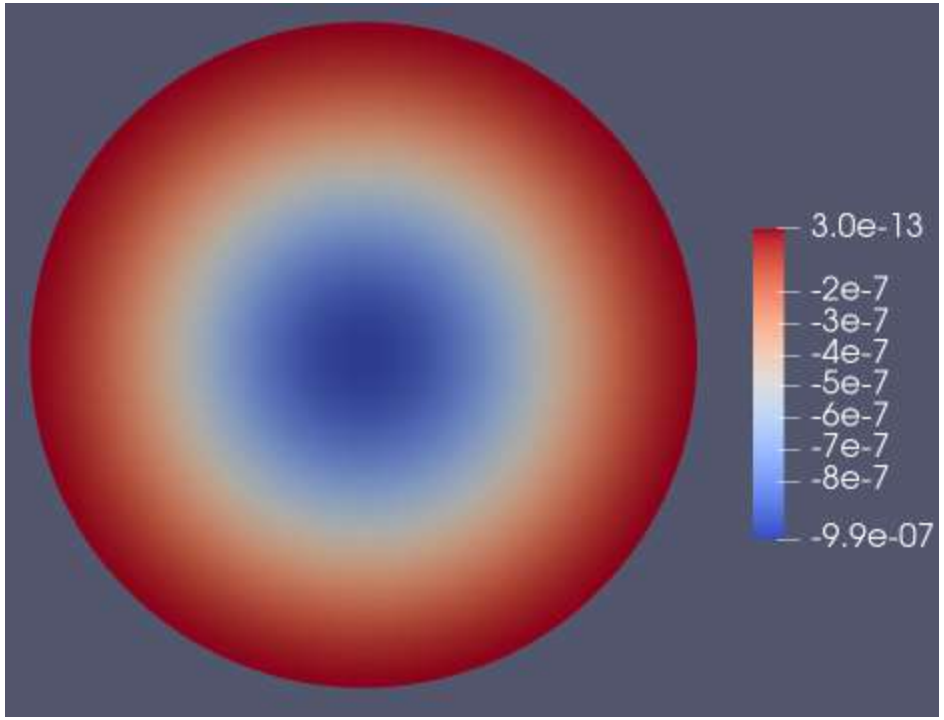


Theoretical solution

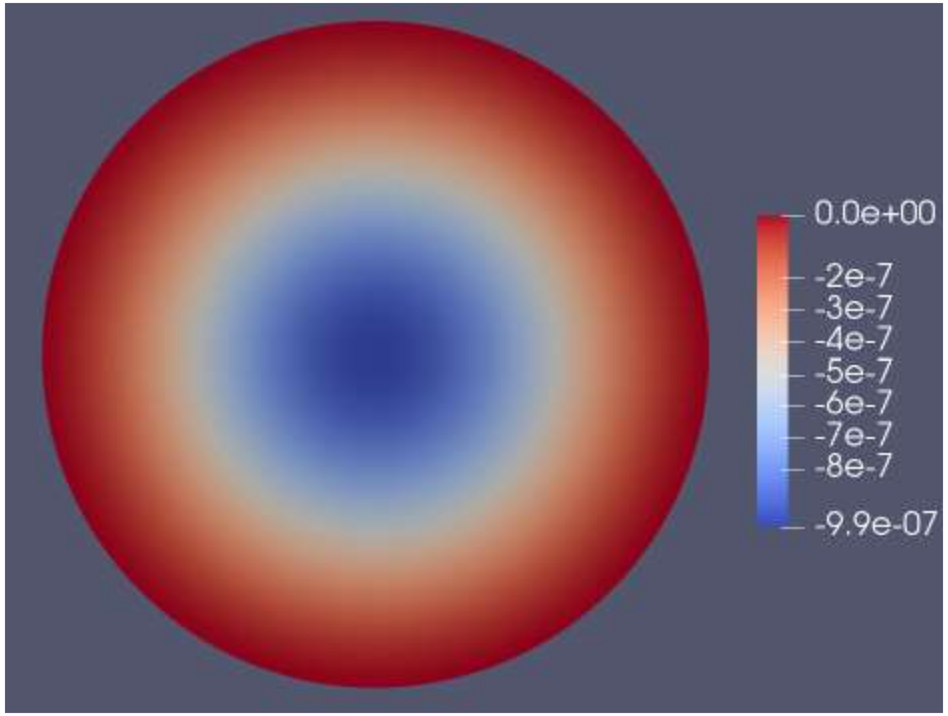


Z-direction displacement distribution Radius: 0.35m (unknown shape)

PINNs

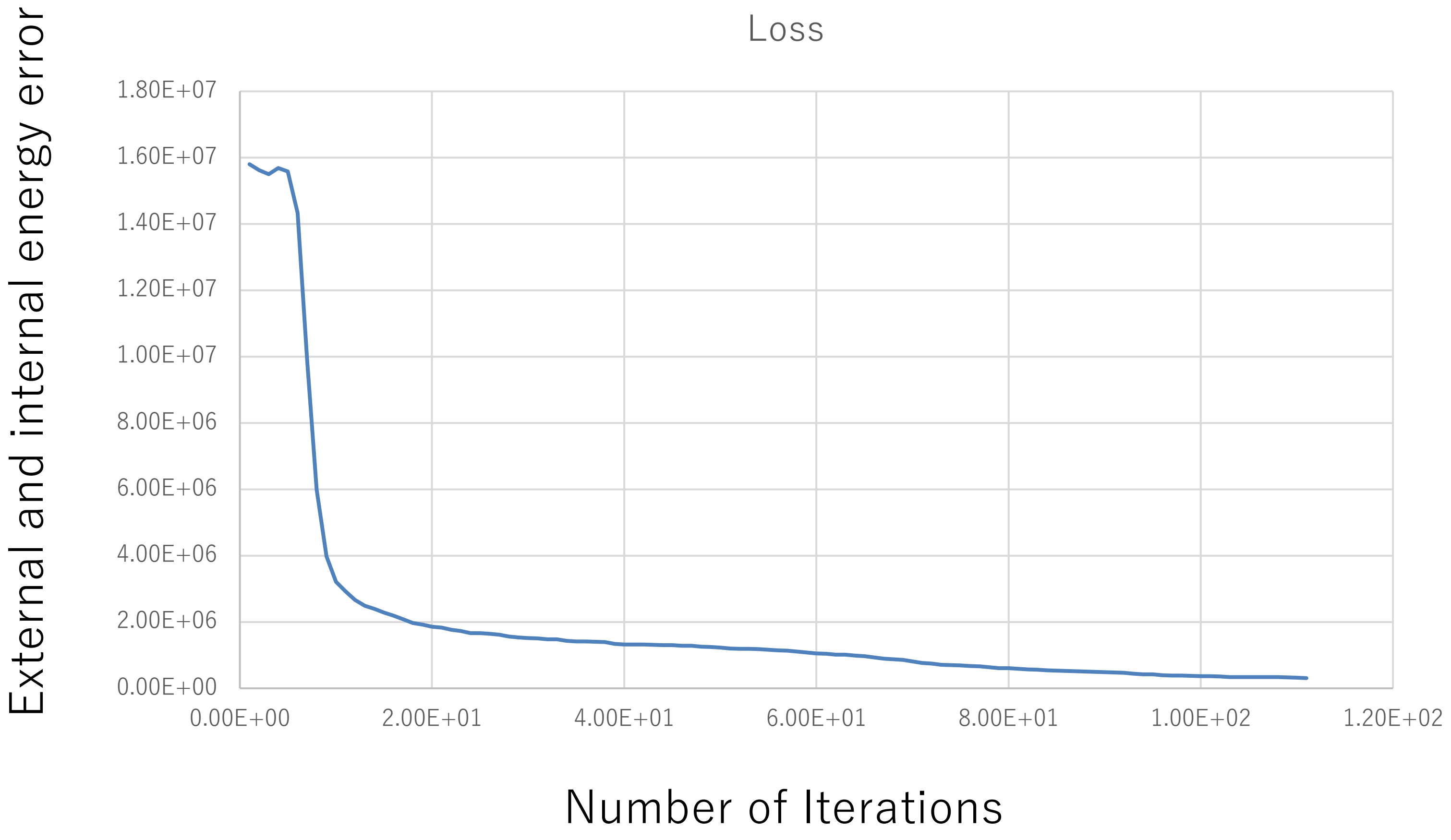


Theoretical solution



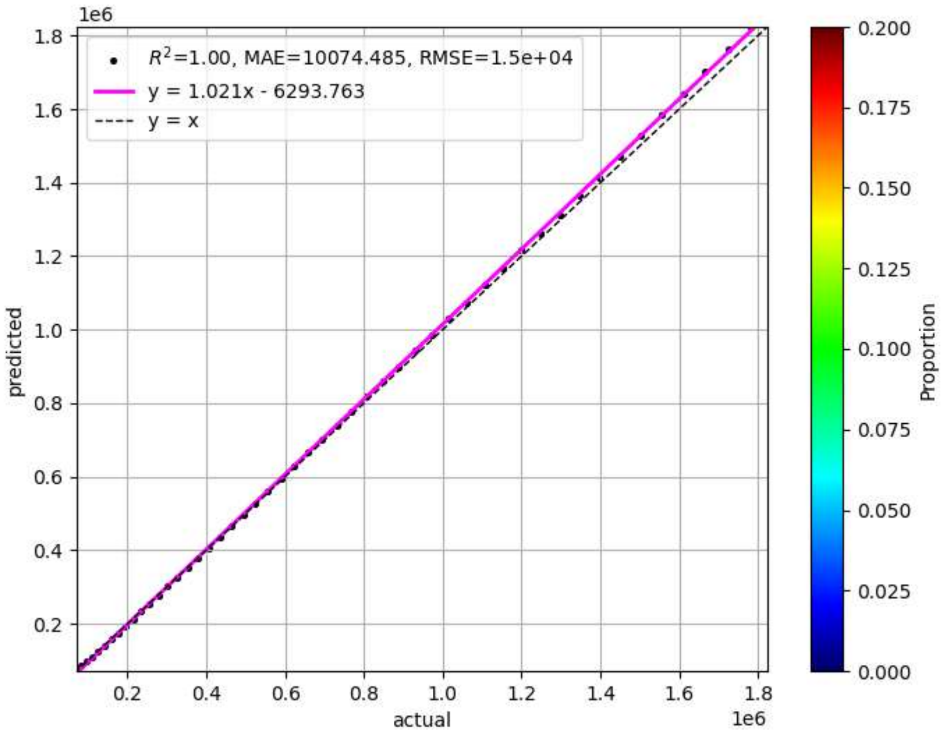
Stress prediction model verification

Convergence of loss function

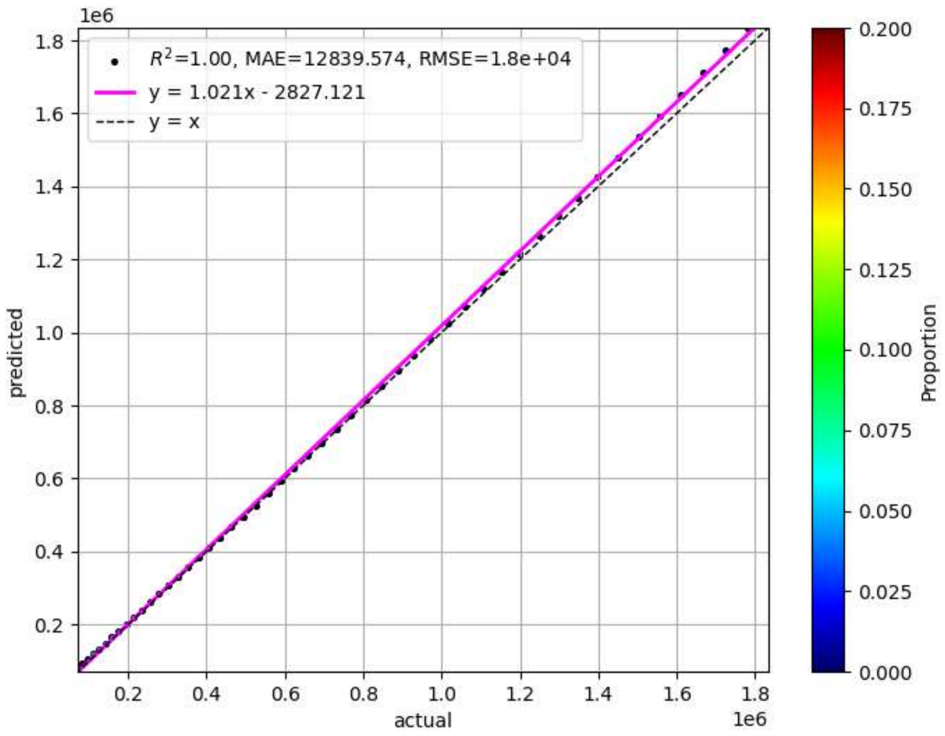


Maximum von Mises stress accuracy comparison plot

Bottom

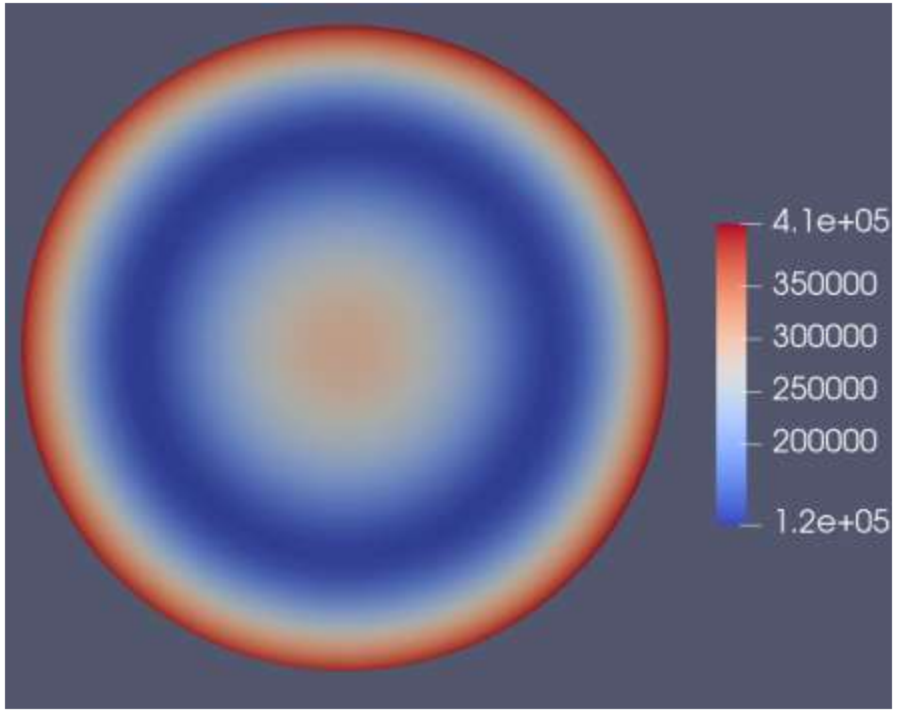


Top

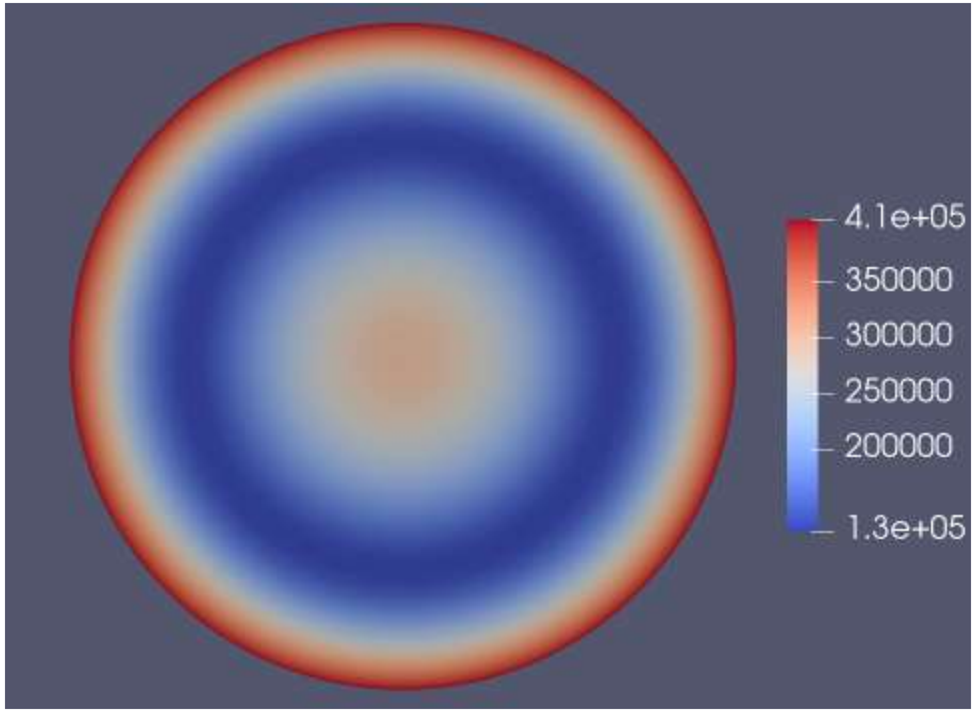


Comparison of von Mises stress (Top) Radius: 0.2387755m (known shape)

PINNs

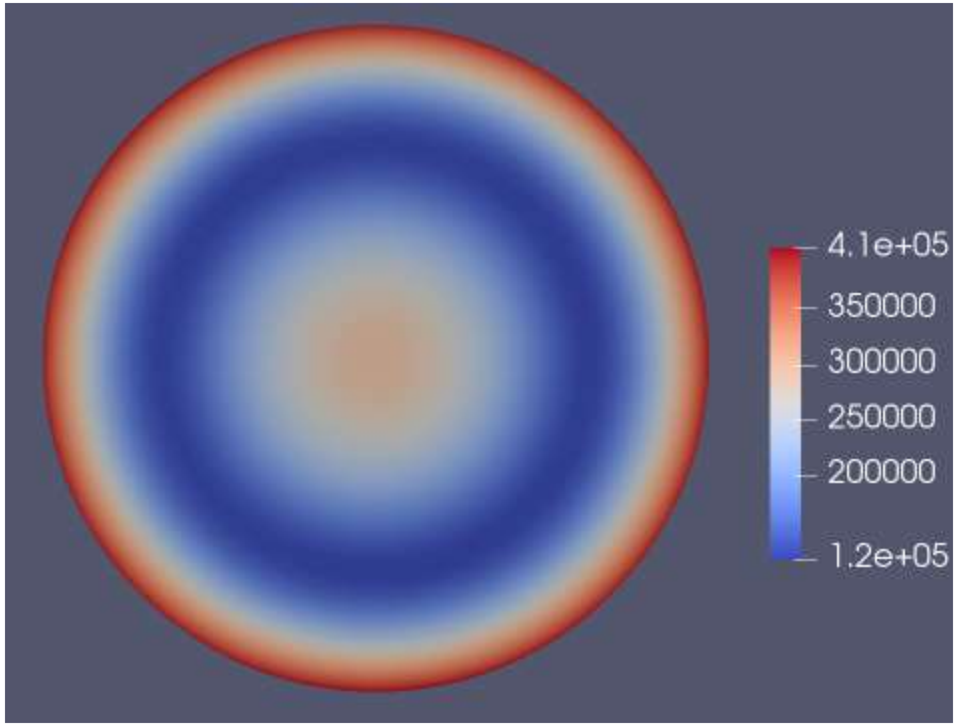


Theoretical solution

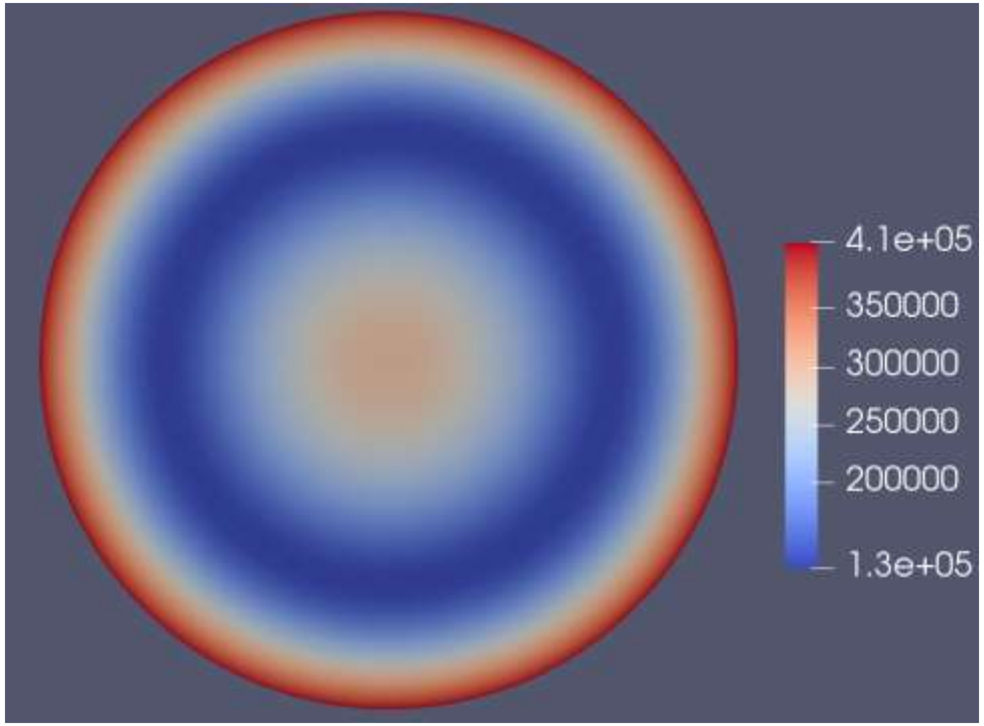


Comparison of von Mises stress (Bottom)
Radius: 0.2387755m (known shape)

PINNs

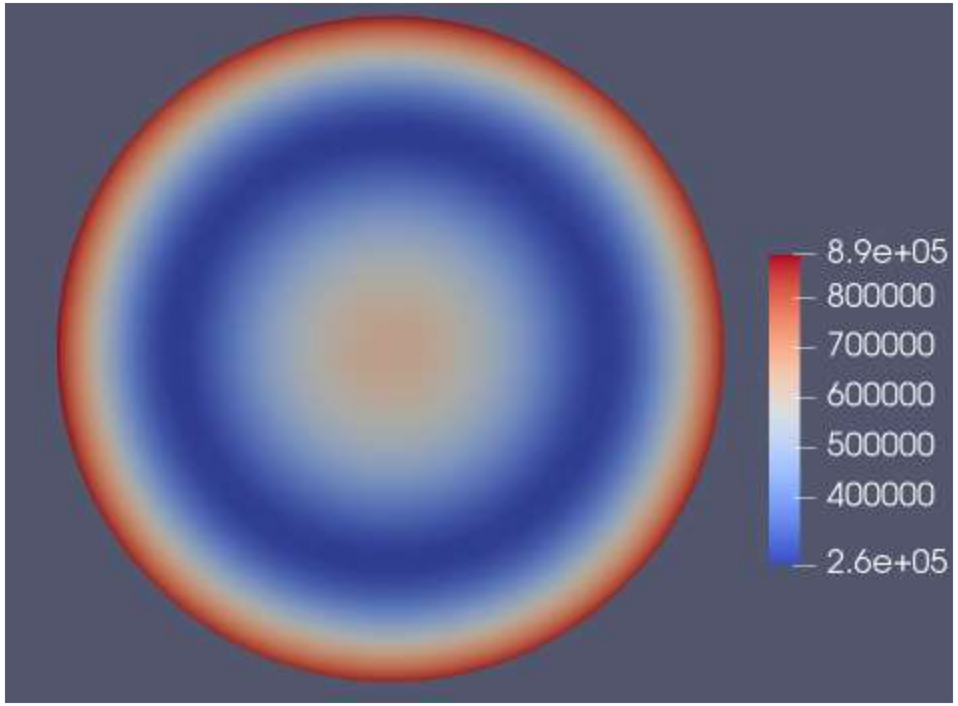


Theoretical solution

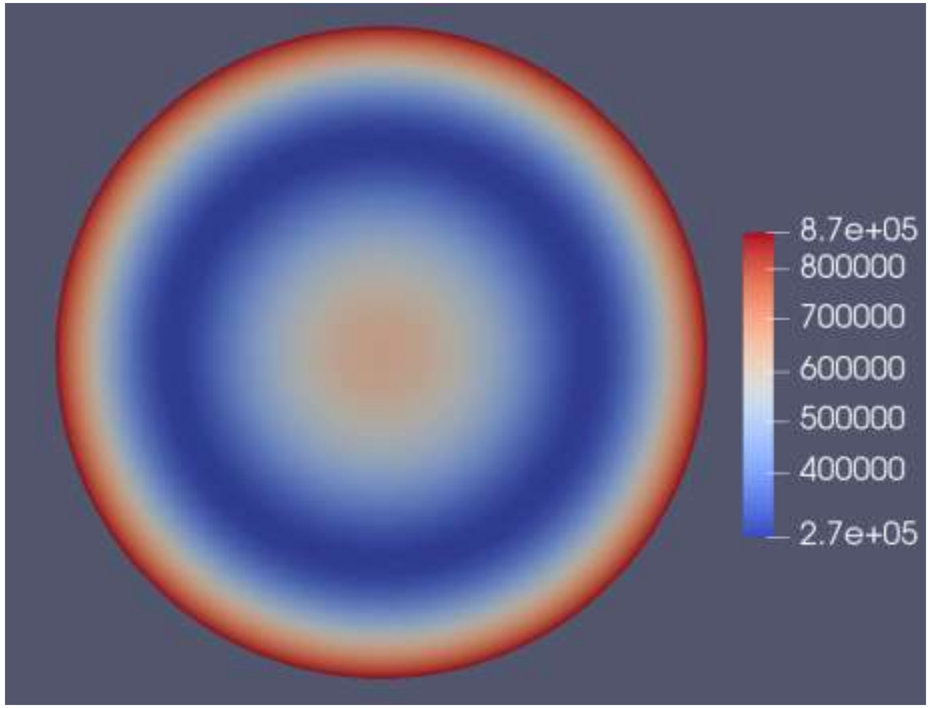


Comparison of von Mises stress (Top) Radius: 0.35m (unknown shape)

PINNs

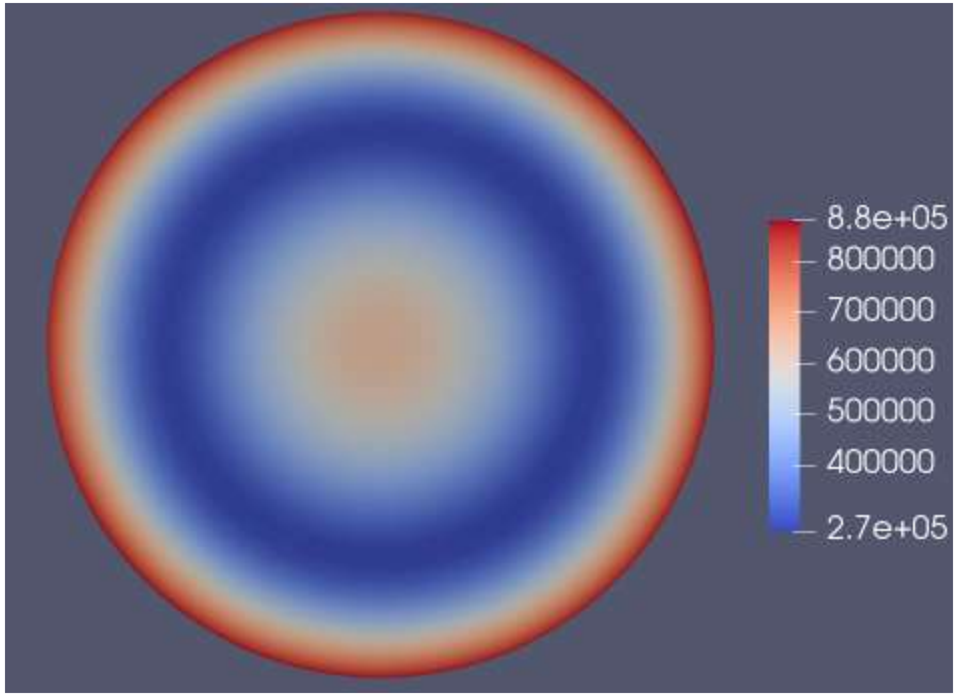


Theoretical solution

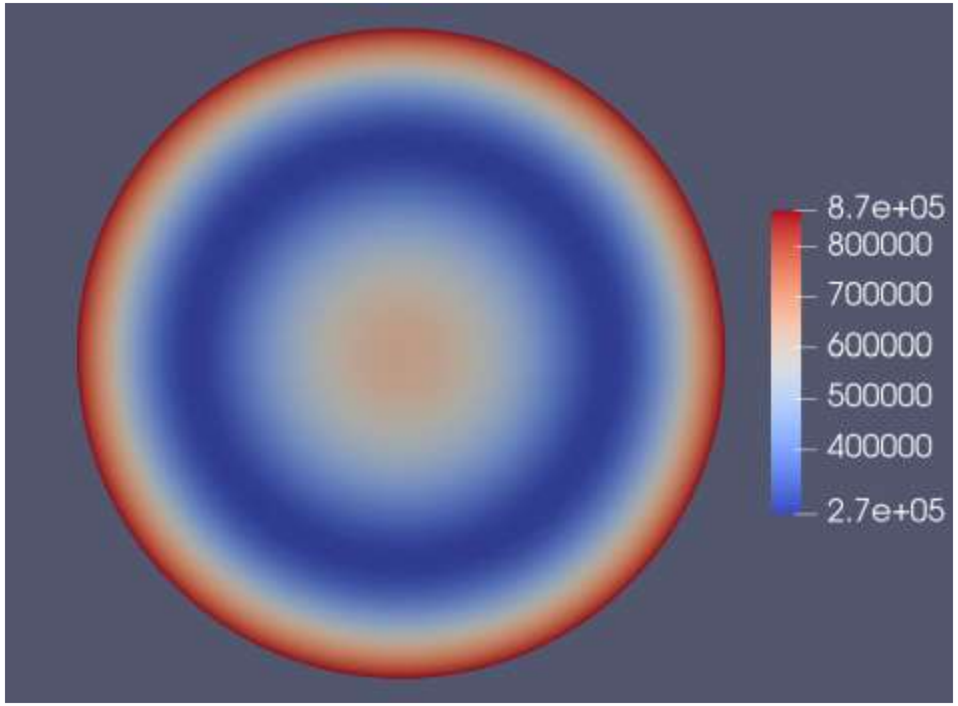


Comparison of von Mises stress (Bottom) Radius: 0.35m (unknown shape)

PINNs



Theoretical solution

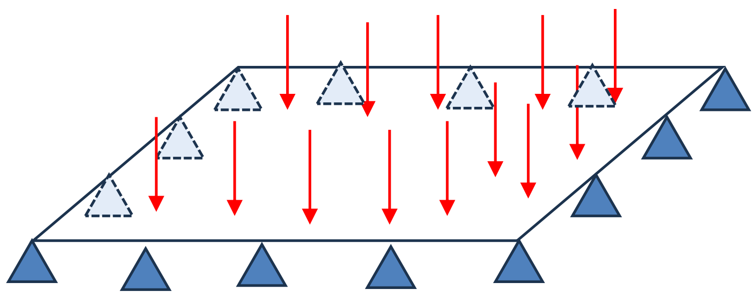


Rectangular Parametric Training

Outline

- The shell element PINNs model was verified using a rectangular model.
- The rectangular model is a rectangle as shown on the next page, and all of the outer edges are fixed. The deflection and stress values are calculated when a uniformly distributed load is applied to it.
- In the training, 100 rectangular models with widths and lengths randomly selected from the range of 0.2 m to 1.0 m were used. This aimed to create a parametric model that could handle any shape within this range. The FEM solution of the same model was also referenced during training.
- The predicted results were compared with the FEM solution, and excellent agreement was observed for both displacement and stress.

Verification model



E = 10.92e11N/m^2
ν = 0.3
q = 6.895e6 N/ m^2
Thickness = 0.0254m

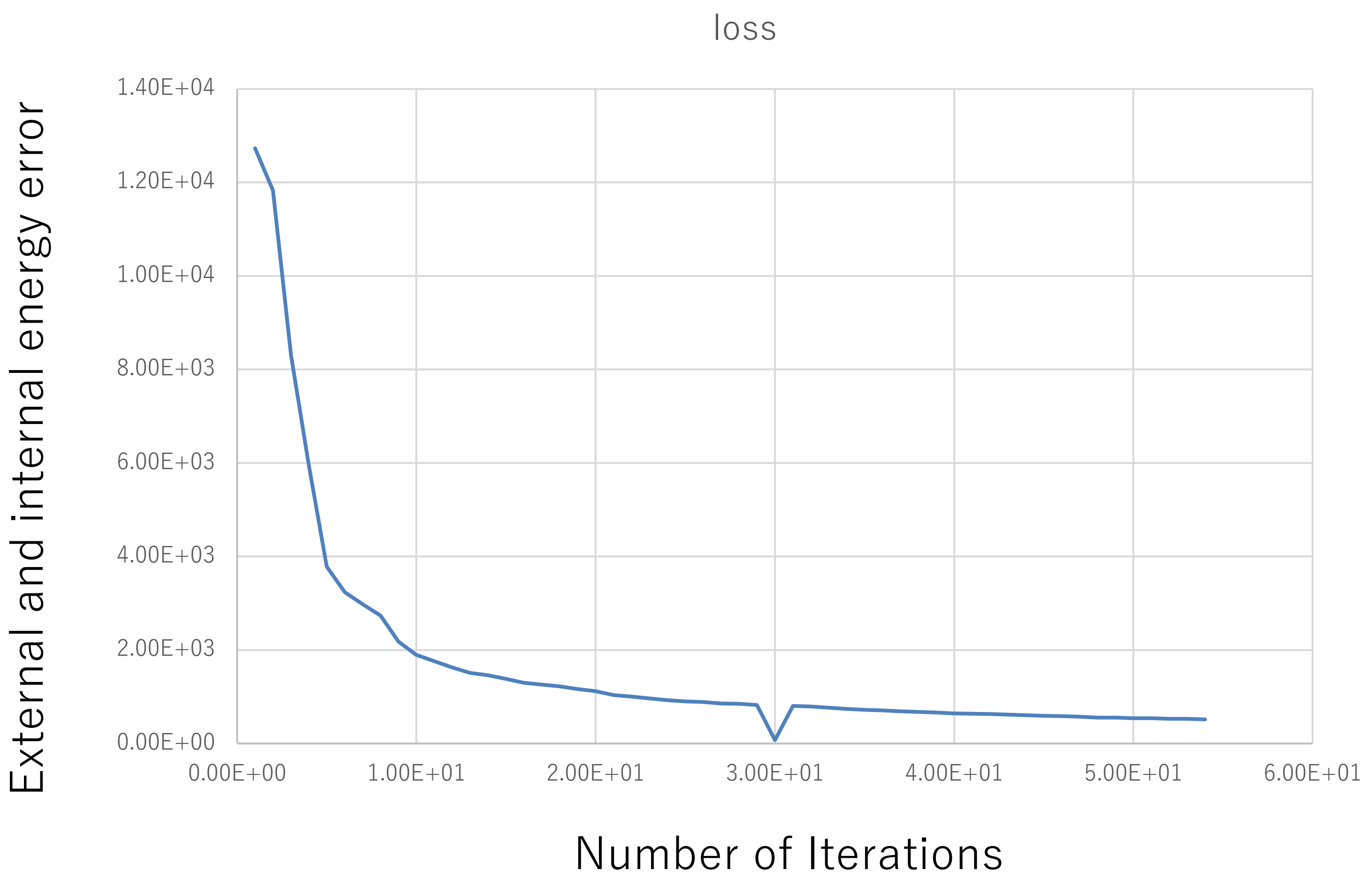
The rectangular plate is fixed at all boundary edges and a distributed load of 6.895e3 N/m^2 is applied to the plate.

Young's modulus: E = 10.92e11N/m^2,
Poisson's ratio: 0.3

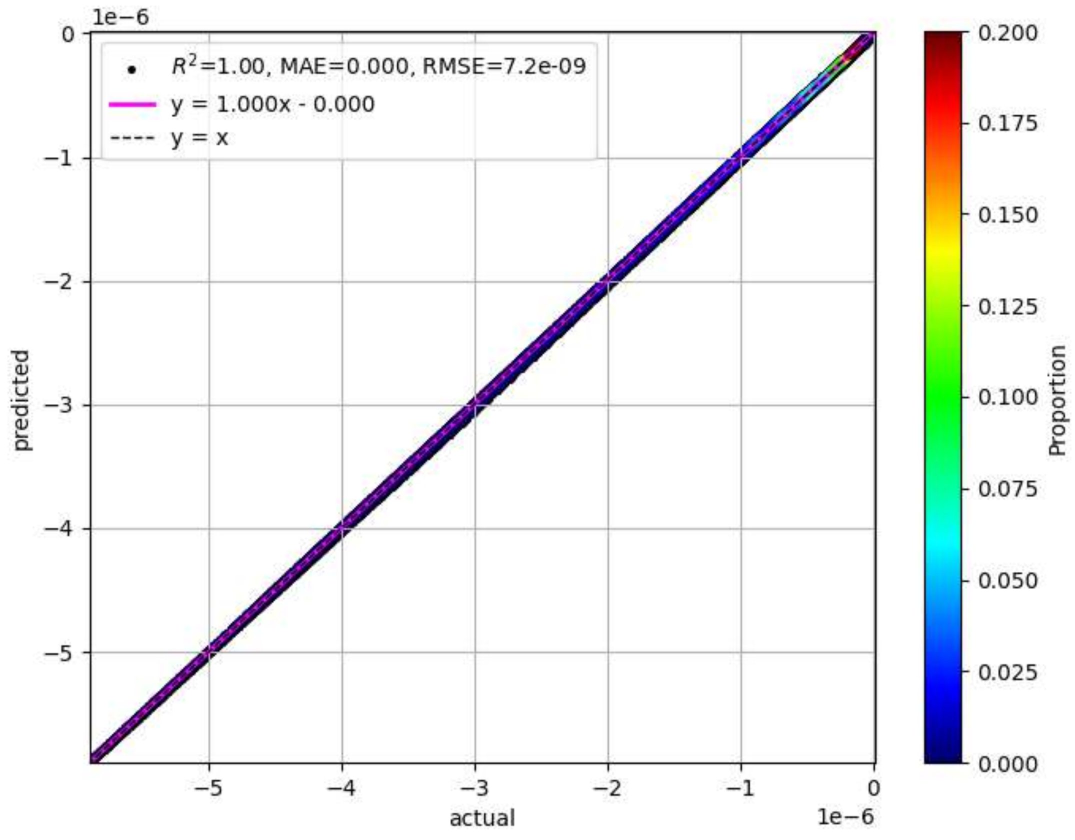
The thickness of the plate is fixed at 0.0254m, and the width and height of the rectangular plate are randomly selected from 0.2 to 1m.
The thick shell theory is applied.

Displacement prediction model verification

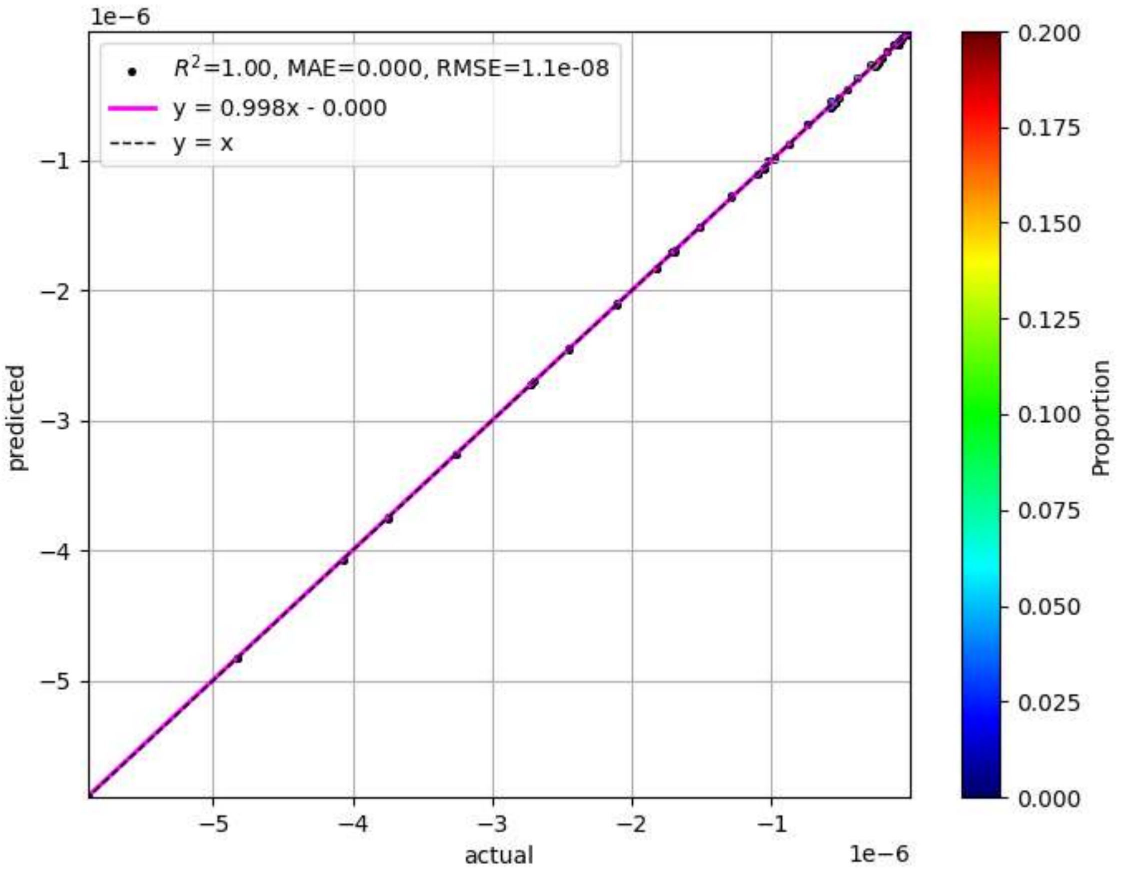
Convergence of loss function



Z Displacement Precision Plot

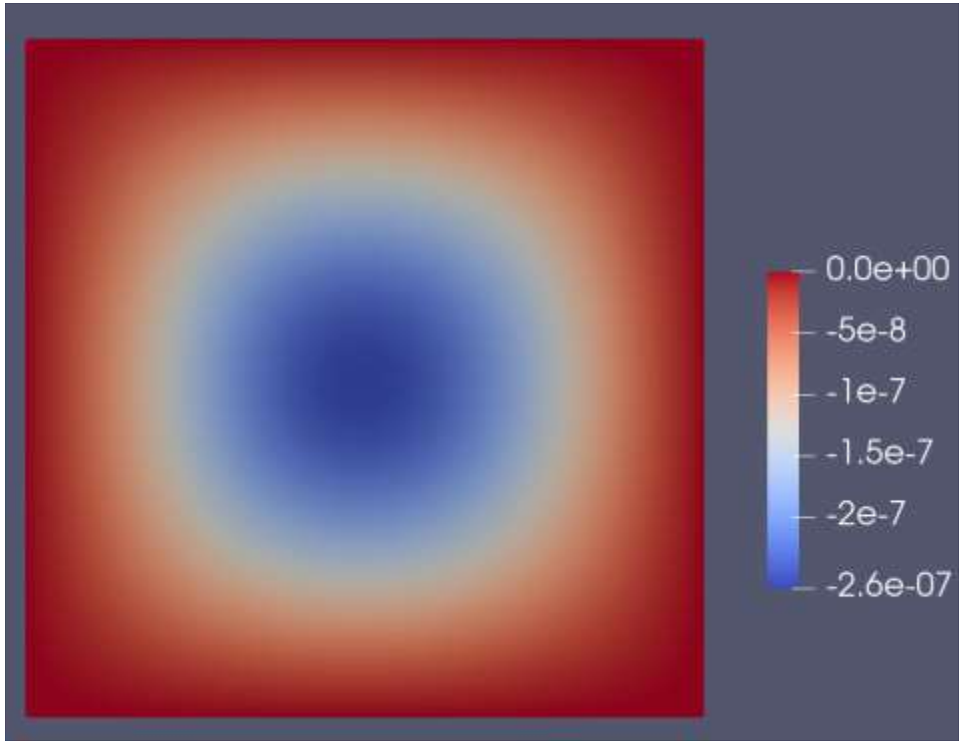


Z-displacement precision plot at the center of the disk (x=0, y=0)

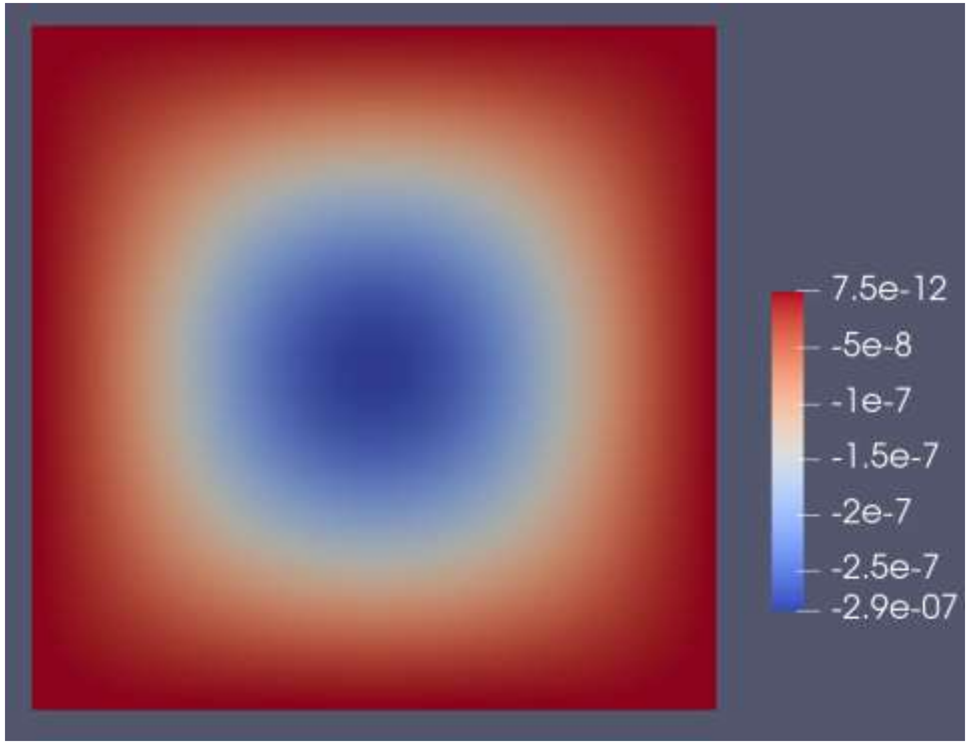


Z-direction displacement distribution
Width: 0.466m Length: 0.466m (known shape)

PINNs

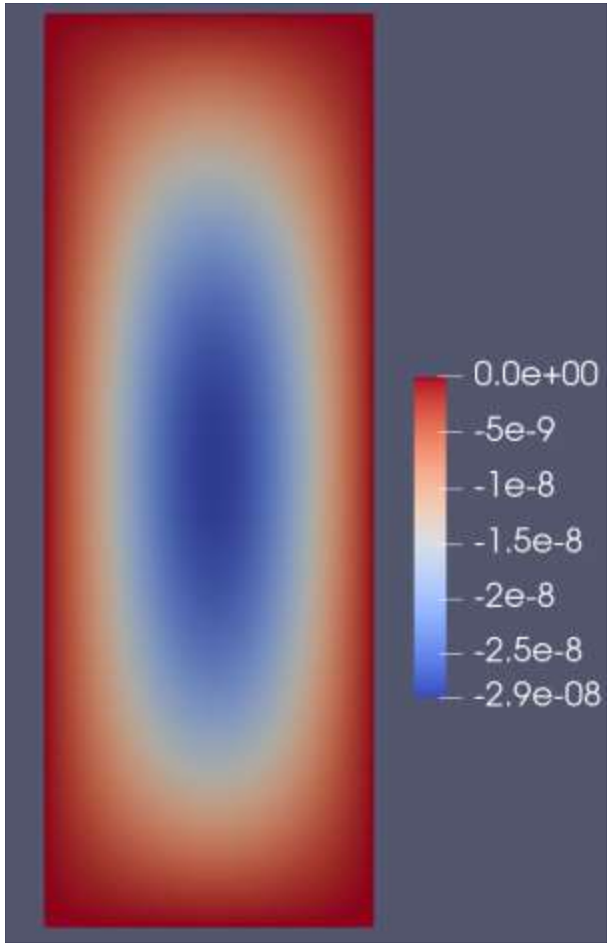


FEM

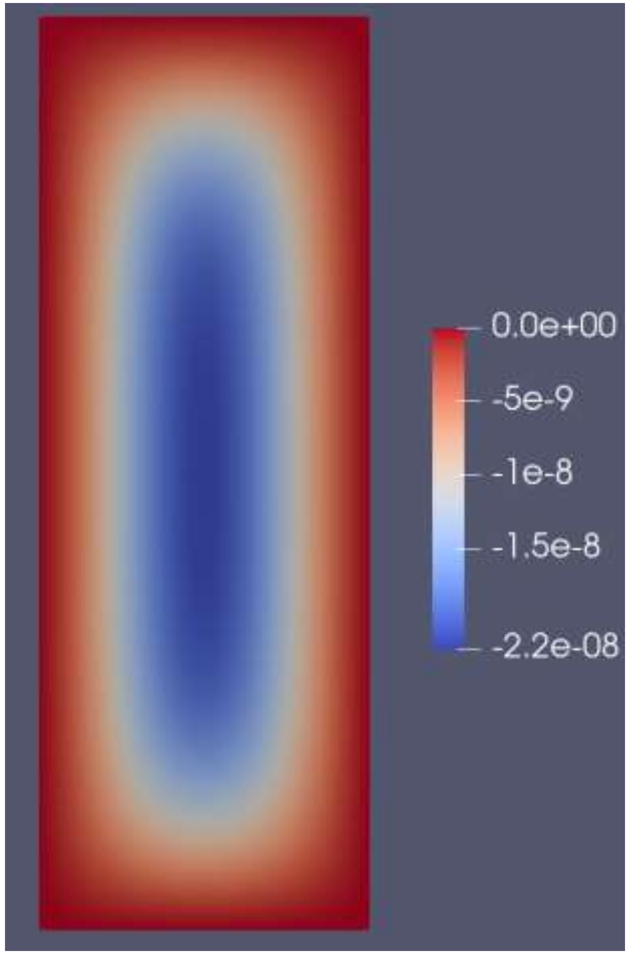


Z-direction displacement distribution
Width: 0.545m Length: 0.2m (known shape)

PINNs

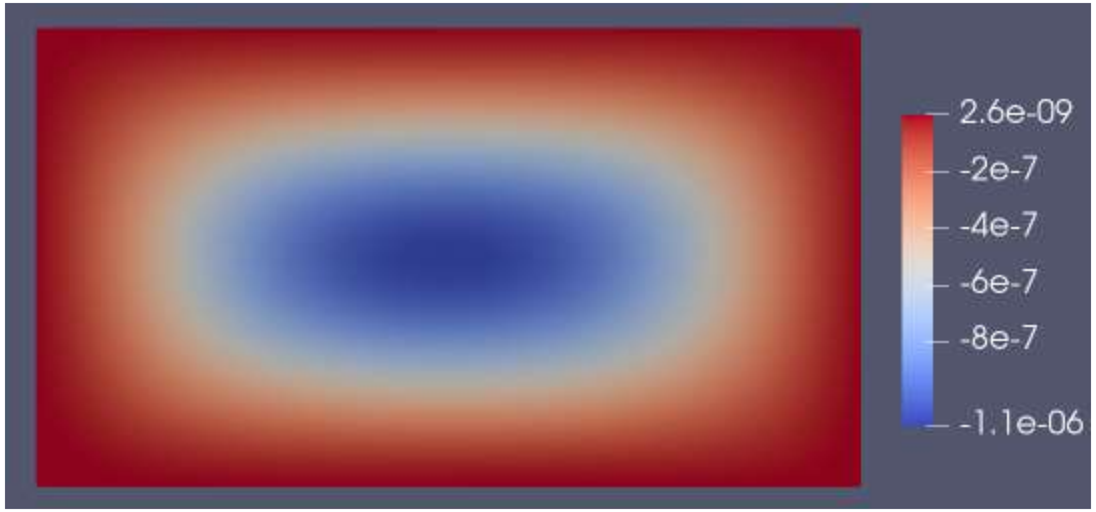


FEM

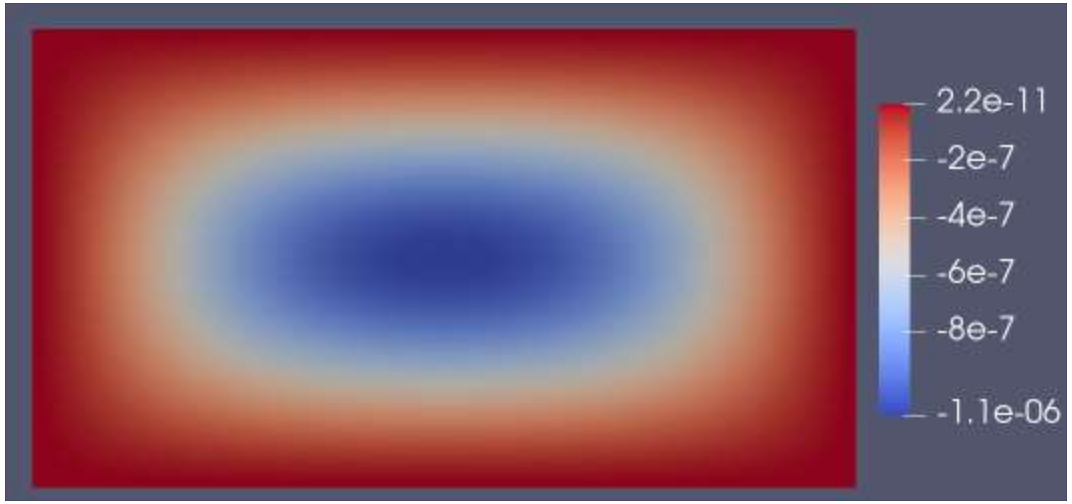


Z-direction displacement distribution
Width: 0.545m Length: 1m (known shape)

PINNs

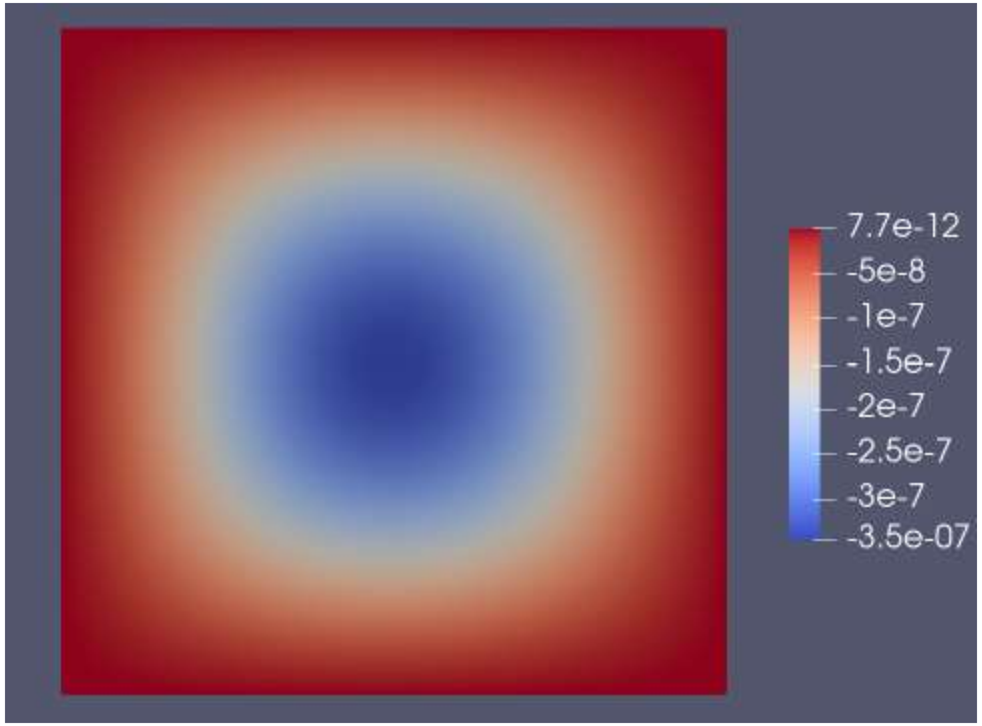


FEM

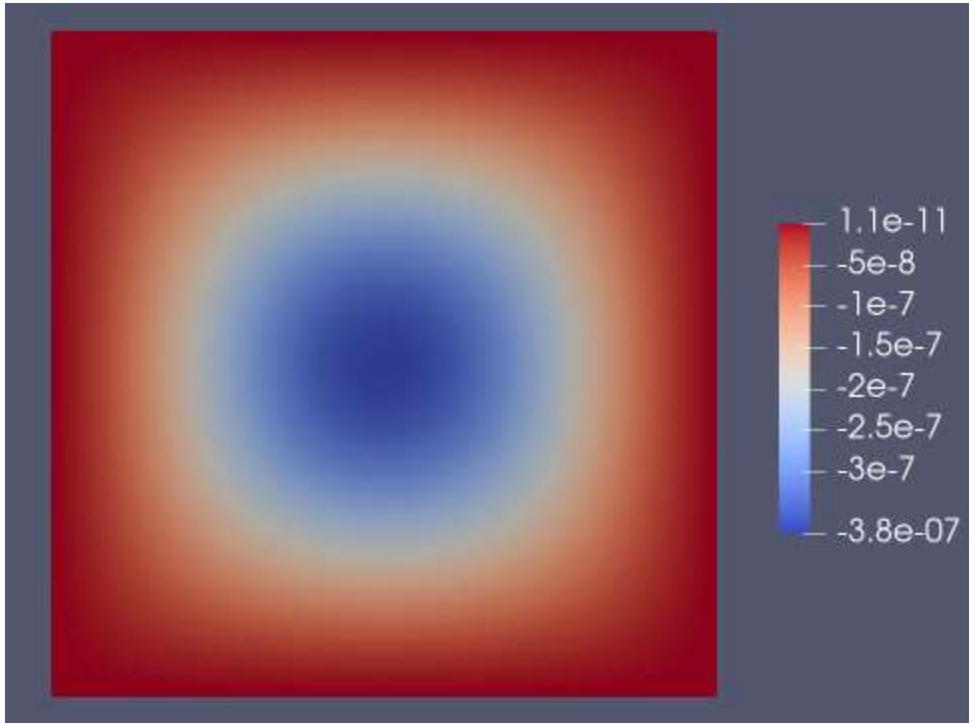


Z-direction displacement distribution
Width: 0.5m Length: 0.5m (unknown shape)

PINNs

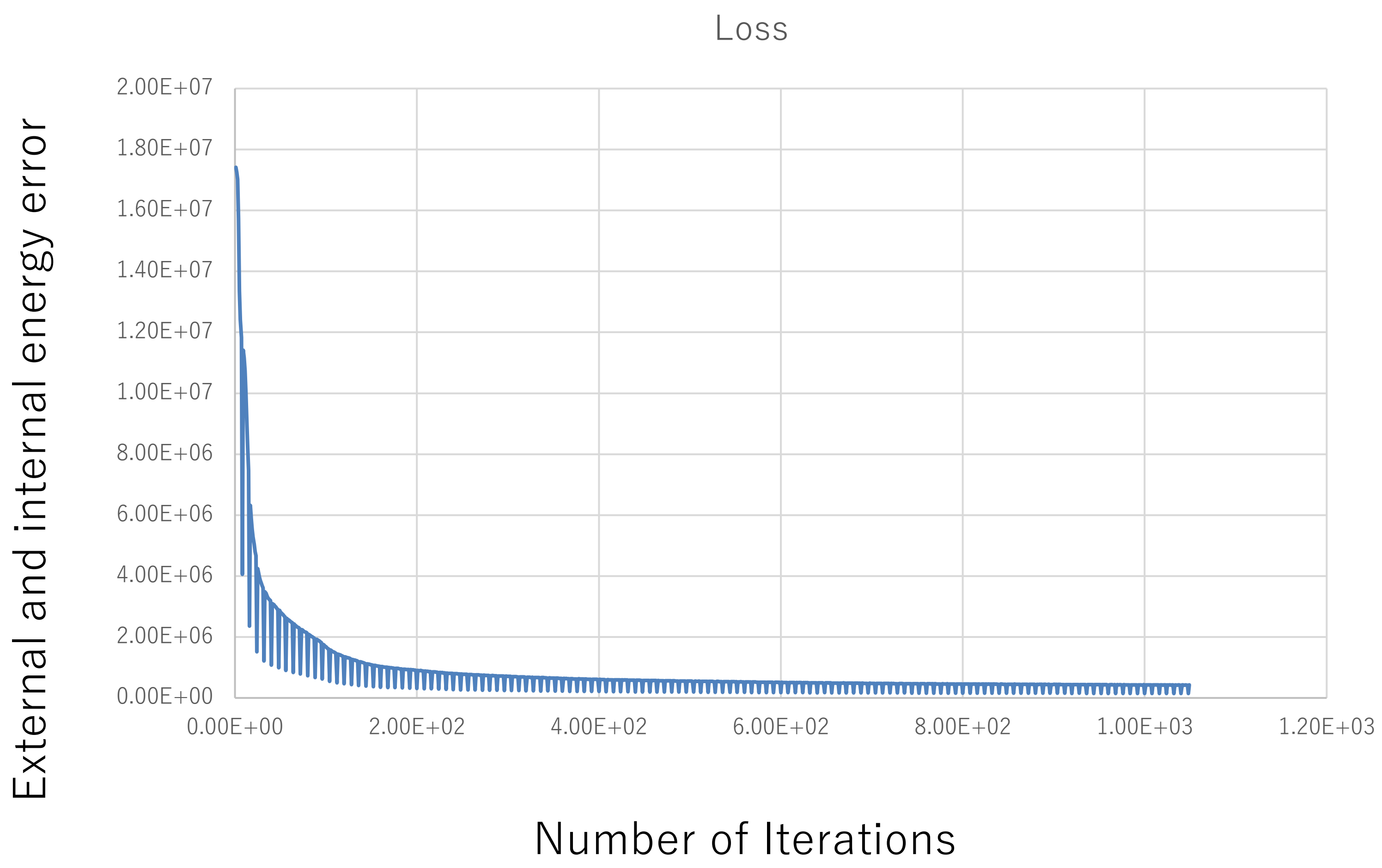


FEM



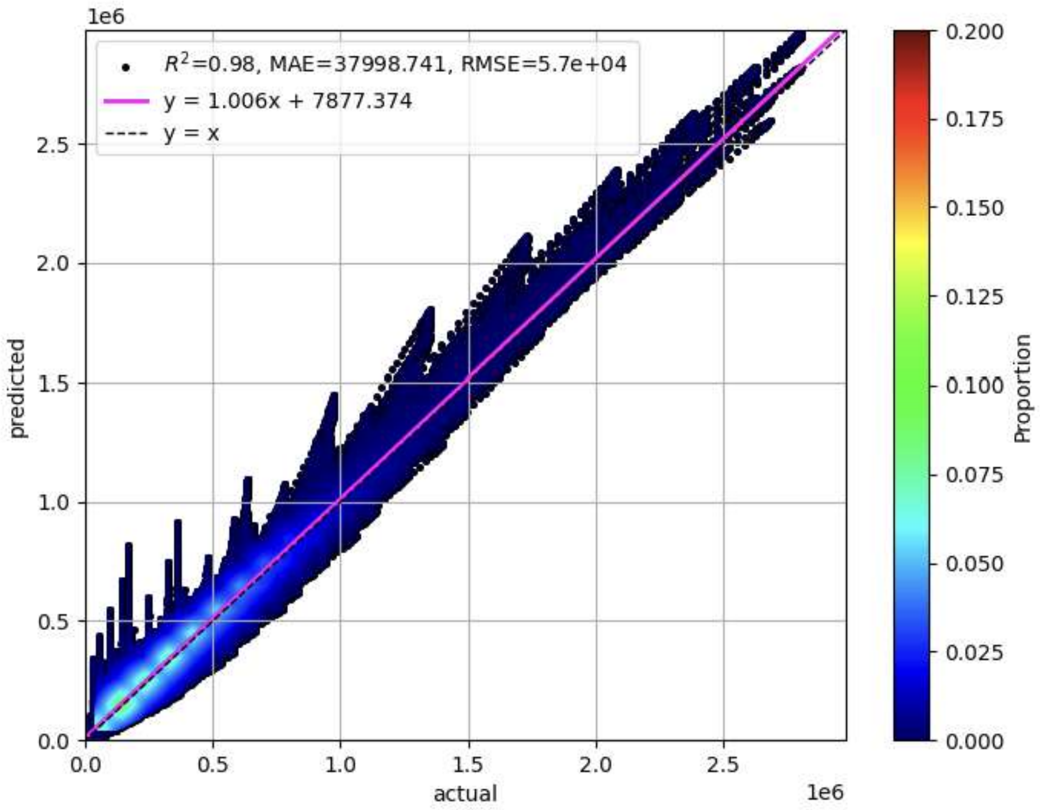
Stress prediction model verification

Convergence of loss function

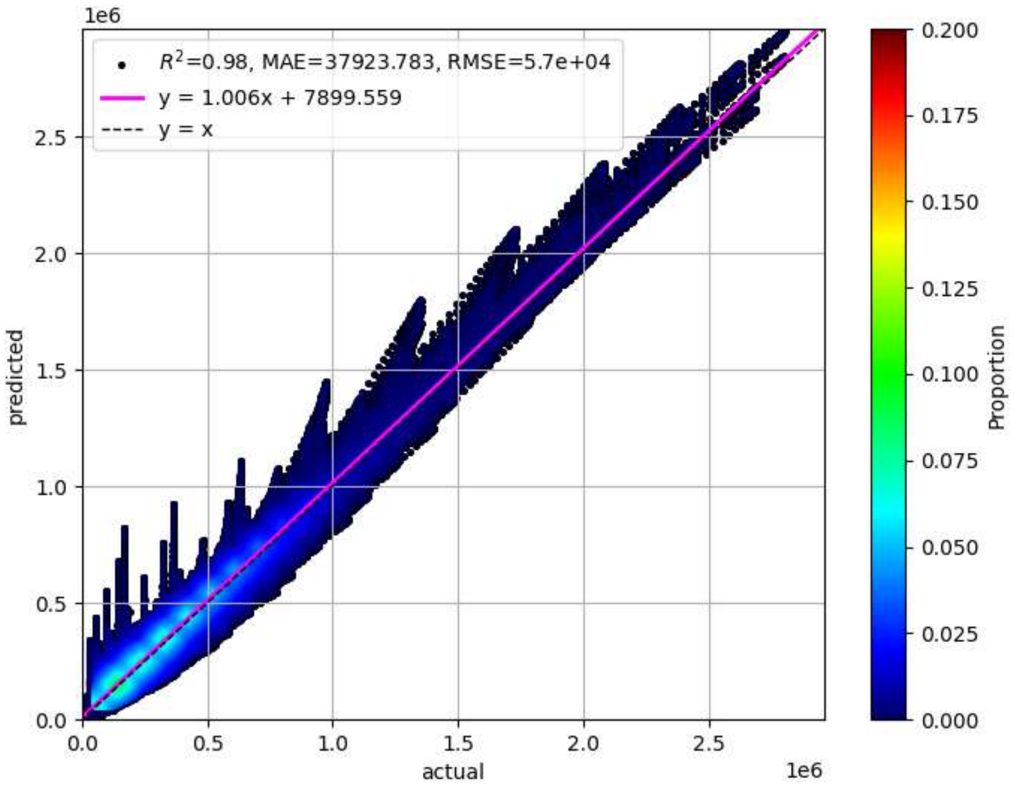


Mises stress accuracy comparison plot

Bottom

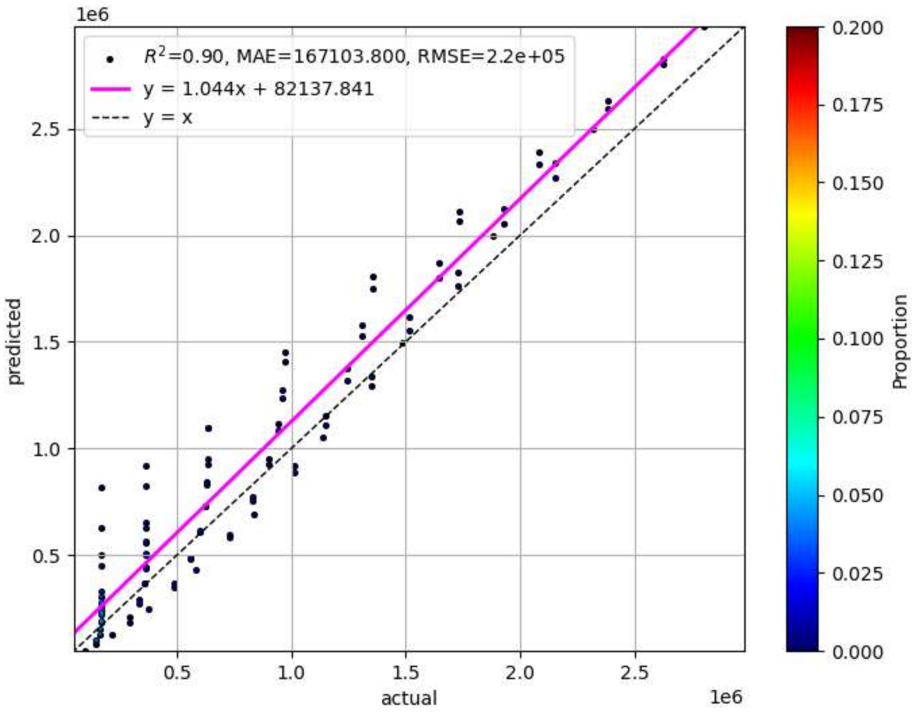


Top

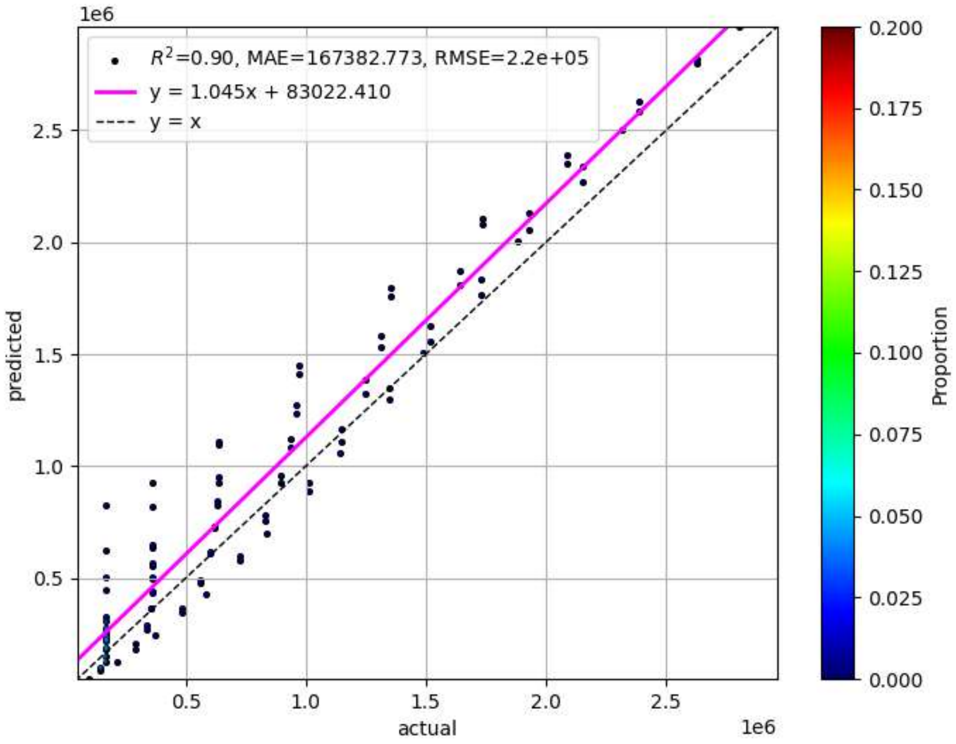


Maximum von Mises stress accuracy comparison plot

Bottom

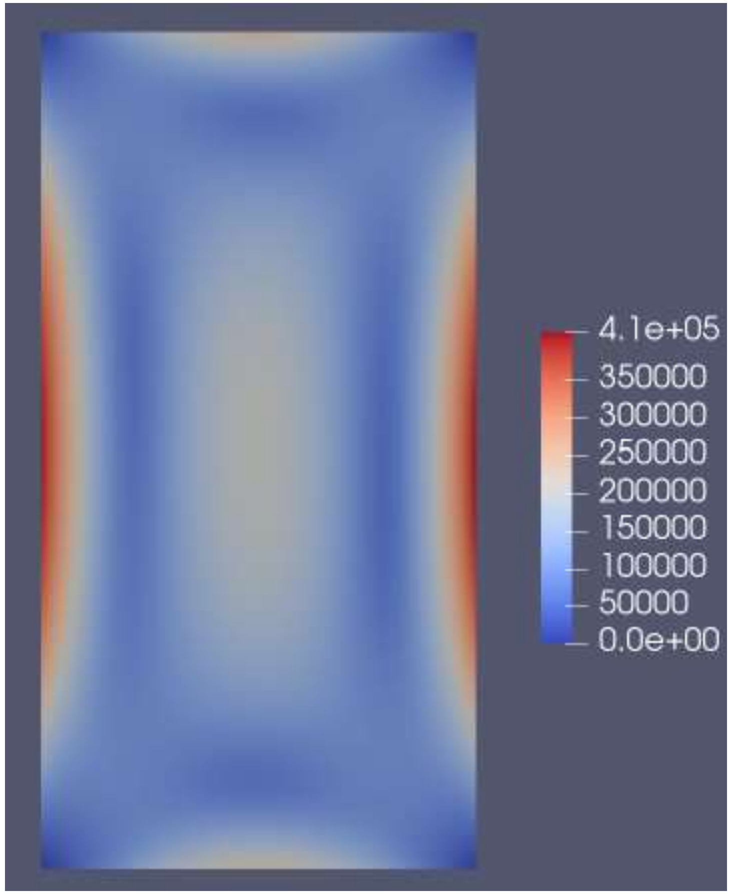


Top

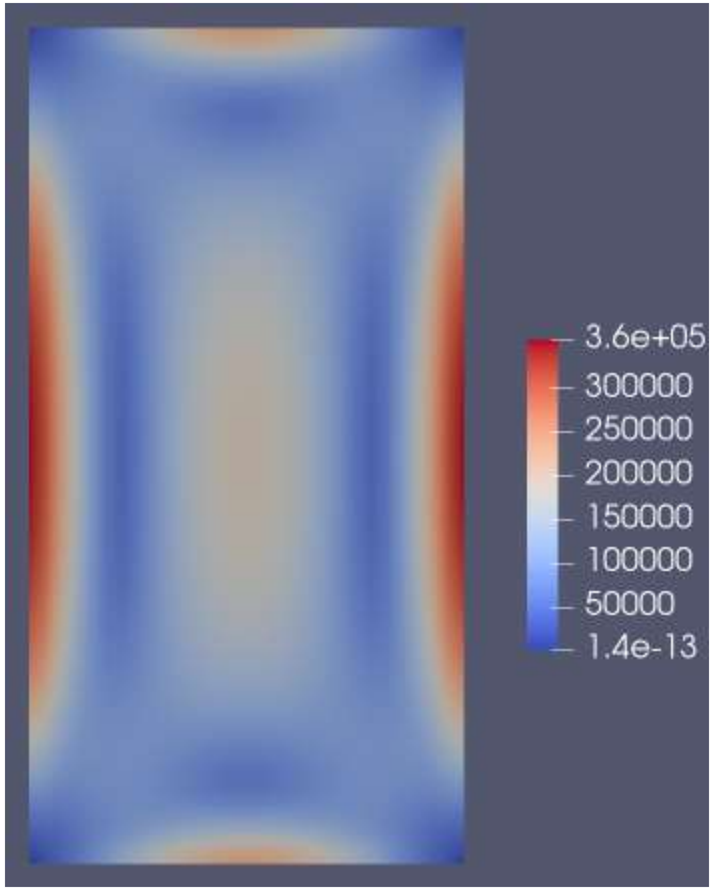


Comparison of von Mises stress (Top)
Width: 0.545m Length: 0.288m (known shape)

PINNs

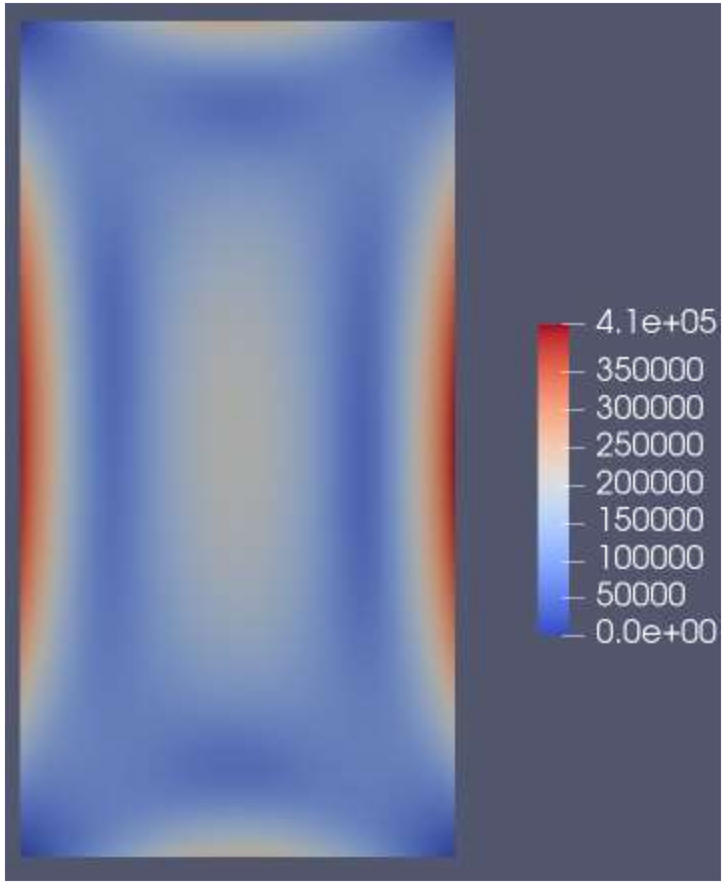


FEM

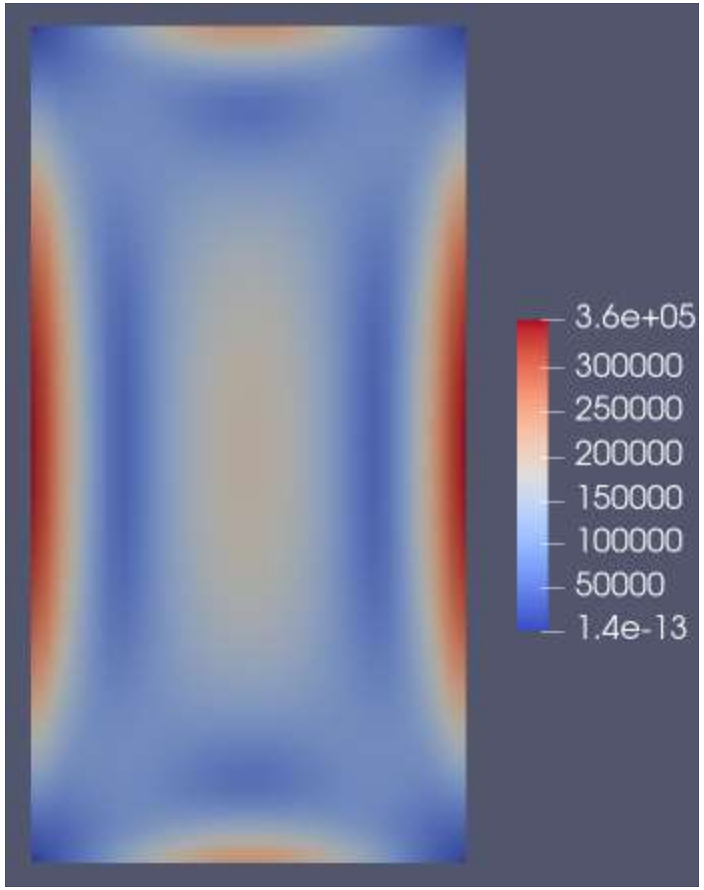


Comparison of von Mises stress (Bottom)
Width: 0.545m Length: 0.288m (known shape)

PINNs

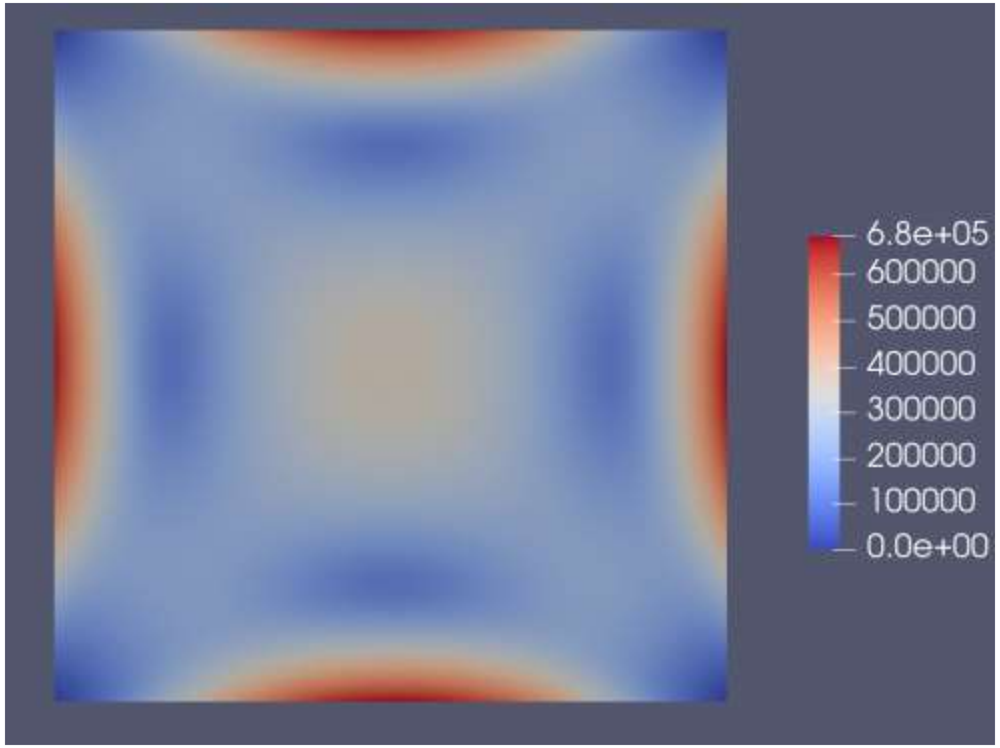


FEM

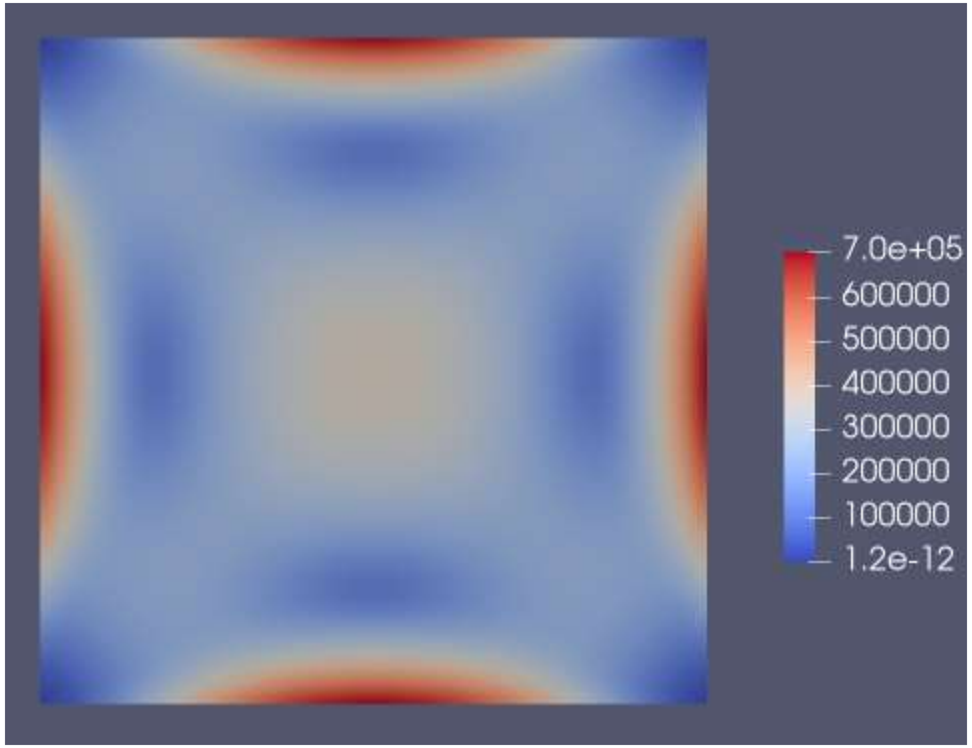


Comparison of von Mises stress (Top)
Width: 0.5m Length: 0.5m (unknown shape)

PINNs

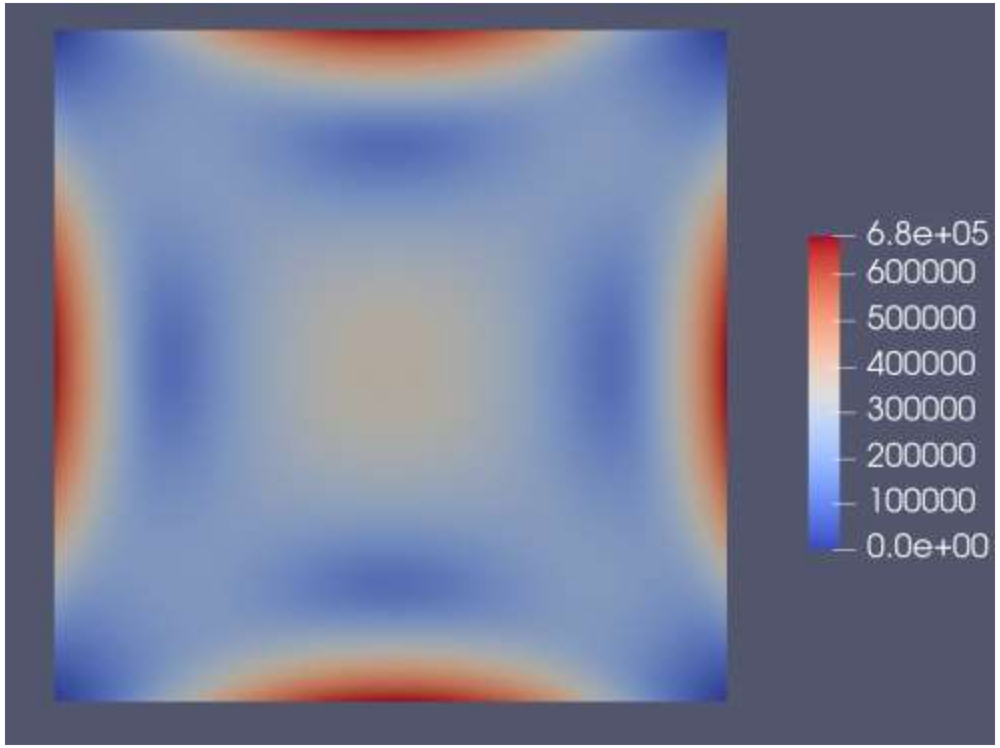


FEM

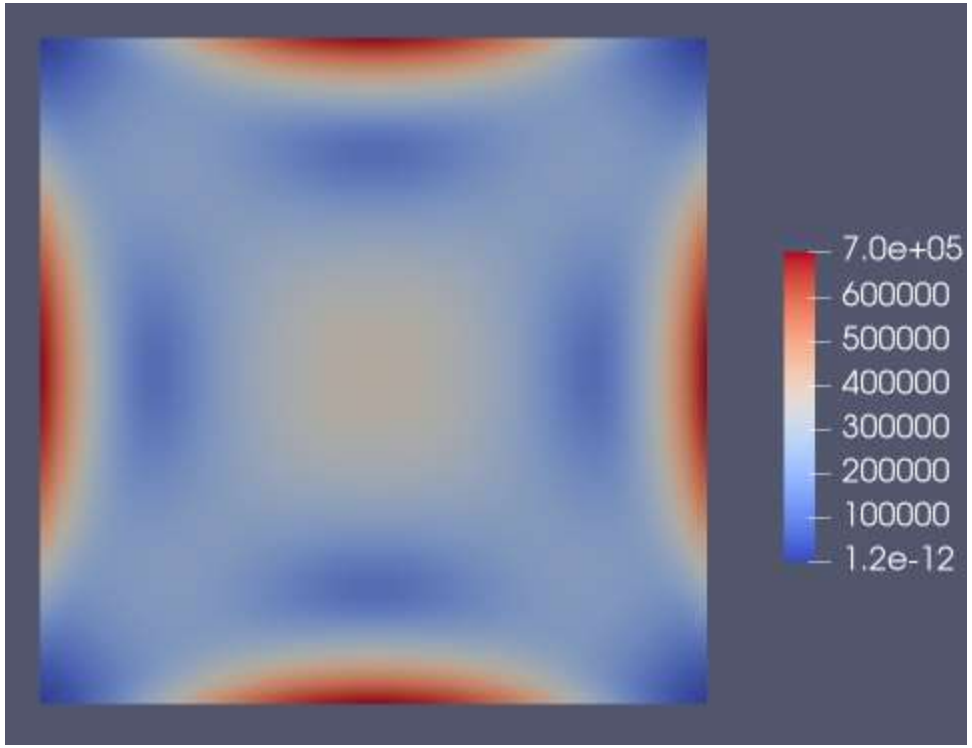


Comparison of von Mises stress (Bottom) Width: 0.5m Length: 0.5m (unknown shape)

PINNs



FEM



End